Prove that each of the following problems is NP-hard.

1 Prove that the following problem is NP-hard: Given an undirected graph G, find *any* integer $k > 374$ such that G has a proper coloring with k colors but G does not have a proper coloring with $k - 374$ colors.

2 A ***bicoloring*** of an undirected graph assigns each vertex a set of *two* colors. There are two types of bicoloring: In a *weak* bicoloring, the endpoints of each edge must use *different* sets of colors; however, these two sets may share one color. In a *strong* bicoloring, the endpoints of each edge must use *distinct* sets of colors; that is, they must use four colors altogether. Every strong bicoloring is also a weak bicoloring.

2.A. Prove that finding the minimum number of colors in a weak bicoloring of a given graph is NP-hard.

2.B. Prove that finding the minimum number of colors in a strong bicoloring of a given graph is NP-hard.

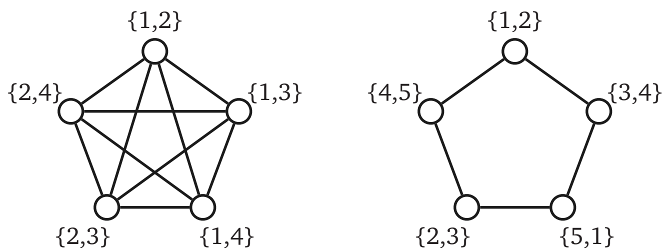

Figure 1: Left: A weak bicoloring of a 5-clique with four colors. Right: A strong bicoloring of a 5-cycle with five colors.