1 Let L be an arbitrary regular language. Prove that the language $\mathit{reverse}(L) := \{w^R \mid w \in L\}$ is regular. *Hint:* Consider a DFA M that accepts L and construct a NFA that accepts $\mathit{reverse}(L)$.

2 Let L be an arbitrary regular language. Prove that the language $\mathit{insert1}(L) := \{x1y \mid xy \in L\}$ is regular.

Intuitively, $\mathit{insert1}(L)$ is the set of all strings that can be obtained from strings in L by inserting exactly one 1. For example, if $L = \{\varepsilon, OOK!\}$, then $\mathit{insert1}(L) = \{1, 1OOK!, O1OK!, OO1K!, OOK1!, OOK!1\}$.

Work on these later:

3 Let L be an arbitrary regular language. Prove that the language $\mathit{delete1}(L) := \{xy \mid x1y \in L\}$ is regular.

Intuitively, $\mathit{delete1}(L)$ is the set of all strings that can be obtained from strings in L by deleting exactly one 1. For example, if $L = \{101101, 00, \varepsilon\}$, then $\mathit{delete1}(L) = \{01101, 10101, 10110\}$.

4 Consider the following recursively defined function on strings:

$$\mathit{stutter}(w) := \begin{cases} \varepsilon & \text{if } w = \varepsilon \\ aa \bullet \mathit{stutter}(x) & \text{if } w = ax \text{ for some symbol } a \text{ and some string } x \end{cases}$$

Intuitively, $\mathit{stutter}(w)$ doubles every symbol in w. For example:

- $\mathit{stutter}(PRESTO) = PPRREESSTTOO$
- $\mathit{stutter}(HOCUS\diamond POCUS) = HHOOCCUUSS\diamond\diamond PPOOCCUUSS$

Let L be an arbitrary regular language.

1. Prove that the language $\mathit{stutter}^{-1}(L) := \{w \mid \mathit{stutter}(w) \in L\}$ is regular.
2. Prove that the language $\mathit{stutter}(L) := \{\mathit{stutter}(w) \mid w \in L\}$ is regular.

5 Consider the following recursively defined function on strings:

$$\mathit{evens}(w) := \begin{cases} \varepsilon & \text{if } w = \varepsilon \\ \varepsilon & \text{if } w = a \text{ for some symbol } a \\ b \cdot \mathit{evens}(x) & \text{if } w = abx \text{ for some symbols } a \text{ and } b \text{ and some string } x \end{cases}$$

Intuitively, $\mathit{evens}(w)$ skips over every other symbol in w. For example:

- $\mathit{evens}(EXPELLIARMUS) = XELAMS$
- $\mathit{evens}(AVADA\diamond KEDAVRA) = VD\diamond EAR$.

Once again, let L be an arbitrary regular language.

1. Prove that the language $\mathit{evens}^{-1}(L) := \{w \mid \mathit{evens}(w) \in L\}$ is regular.
2. Prove that the language $\mathit{evens}(L) := \{\mathit{evens}(w) \mid w \in L\}$ is regular.