CS/ECE 374 A (Spring 2022)
Homework 2 (due Feb 3 Thursday at 10am)

Instructions: As in Homework 1.

Problem 2.1: For each of the following languages, give a regular expression that describes that language, and briefly argue why your expression is correct. Below, $\#_0(x)$ denotes the number of 0's in x, and $\#_1(x)$ denotes the number of 1's in x.

(a) All strings in $\{0,1\}^*$ of length at least 5 such that the symbols at positions divisible by 5 are all 0's. For example: the string 1001**0**1111**0**0101**0**11 is in the language because the bold symbols are all 0's.

(b) All strings $x \in \{0,1\}^*$ such that x does not begin with 101 and $\#_0(x)$ is divisible by 3.

(c) All strings in $\{0,1\}^*$ that do not contain 0111 as a substring.

(d) All strings in $\{0,1,2\}^*$ that have at least four 0's and exactly two 1's.

Problem 2.2: Describe a DFA that accepts each of the following languages. Describe briefly what each state in your DFA *means*. For (a)–(c), you should draw the complete DFA. For (d), do not attempt to draw your DFA, since the number of states could be huge; instead, give a mathematically precise description of the states Q, the start state s, the accepting states A, and the transition function δ.

(a) (20 pts) All strings in $\{0,1\}^*$ of length at least 5 such that the symbols at positions divisible by 5 are all 0's.

(b) (20 pts) All strings $x \in \{0,1\}^*$ such that x does not begin with 101 and $\#_0(x)$ is divisible by 3.

(c) (20 pts) All strings in $\{0,1\}^*$ that do not contain 011001 as a substring.

(d) (40 pts) All strings $x \in \{0,1\}^*$ such that x contains 0^9 as a substring, or $|x|$ is divisible by 8, or $\#_0(x) - \#_1(x)$ is divisible by 7.