CS/ECE 374 ✧ Spring 2021

Homework 10

Due Thursday, April 29, 2021 at 10am

Groups of up to three people can submit joint solutions. Each problem should be submitted by exactly one person, and the beginning of the homework should clearly state the Gradescope names and email addresses of each group member. In addition, whoever submits the homework must tell Gradescope who their other group members are.

---

1. Recall that $L_u = \{\langle M, w\rangle \mid M \text{ accepts } w\}$ is langauge of a UTM, and $L_{HALT} = \{\langle M\rangle \mid M \text{ halts on blank input}\}$ is the Halting language.

   - Let $L_{374\text{A}} = \{\langle M\rangle \mid M \text{ accepts at least 374 distinct input strings}\}$. Prove that $L_{374\text{A}}$ is undecidable.
   - Prove that $L_u \leq L_{HALT}$
   - **Not to submit:** Prove that $L_{HALT} \leq L_u$.

2. Consider an instance of the Satisfiability Problem, specified by clauses $C_1, \ldots, C_m$ over a set of Boolean variables $x_1, \ldots, x_n$. We say that the instance is *monotone* if each term in each clause consists of a nonnegated variable; that is each term is equal to x_i, for some i, rather than $\bar{x}_i$. Monotone instance of Satisfiability are very easy to solve: They are always satisfiable, by setting each variable equal to 1.

   For example, suppose we have the three clauses

   $$(x_1 \vee x_2), (x_1 \vee x_3), (x_2 \vee x_3)$$

   This is monotone, and indeed the assignment that sets all three variables to 1 satisfies all the clauses. But we can observe that this is not the only satisfying assignment; we could also have set x_1 and x_2 to 1 and x_3 to 0. Indeed, for any monotone instance, it is natural to ask how few variables we need to set to 1 in order to satisfy it.

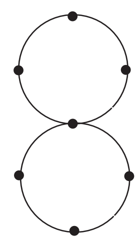
   Given a monotone instance of Satisfiability, together with a number k, the problem of *Monotone Satisfiability with Few True Variables* asks: Is there a satisfying assignment for the instance in which at most k variables are set to 1? Describe a polynomial time reduction from Vertex Cover to this problem. You should also prove the correctness of the reduction.

3. Given an undirected graph $G = (V, E)$, a partition of V into $V_1, V_2, \ldots, V_k$ is said to be a clique cover of size k if each V_i is a clique in G. CLIQUE-COVER is the following decision problem: given G and integer k, does G have a clique cover of size at most k?

   - Describe a polynomial-time reduction from CLIQUE-COVER to SAT. Does this prove that CLIQUE-COVER is NP-Complete? For this part you just need to describe the reduction clearly, no proof of correctness is necessary. *Hint:* Use variablex $x(u, i)$ to indicate that node u is in partition i.
   - Describe a polynomial-time reduction from k-Color to CLIQUE-COVER.

You should also prove the correctness of the reductions.

4. **Not to submit:** We call an undirected graph an *eight-graph* if it has an odd number of nodes, say $2n-1$, and consists of two cycles C_1 and C_2 on n nodes each and C_1 and C_2 share exactly one node. See figure below for an eight-graph on 7 nodes.



Given an undirected graph G and an integer k, the EIGHT problem asks whether or not there exists a subgraph which is an eight-graph on $2k-1$ nodes. Prove that EIGHT is NP-Complete