Review

Thursday, February 20, 2020 3:27 PM

$\boxed{4}$ $\text{scramble}(w) = \begin{cases} w & \text{if } |w| \leq 1 \\ ba \cdot \text{scramble}(x) & \text{if } w = abx, \; a \in \Sigma, b \in \Sigma, x \in \Sigma^* \end{cases}$

Prove that $\text{scramble}(\text{scramble}(w)) = w$

Induction on $|w|$

Base Cases: $|w| \leq 1$

$\text{scramble}(w) = w$

$\text{scramble}(\text{scramble}(w)) = \text{scramble}(w) = w$ ✓

When $|w| \geq 2$..

— $w = abx$ for some a, b, x

$\text{scramble}(w) = ba \cdot \text{scramble}(x)$

$\text{scram}(\text{scram}(w)) = \text{scram}(ba \cdot \text{scram}(x))$

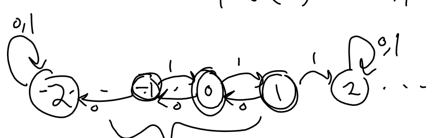
$= ab \cdot \text{scram}(\text{scram}(x))$

IH: $\forall |x| < |w|$, $\underline{\text{scram}(\text{scram}(x)) = x}$

— $= ab \cdot x$ ✓

$\boxed{13}$ "All strings s.t. every prefix has difference of $\#_0$ and $\#_1$ $\leq \pm 1$"

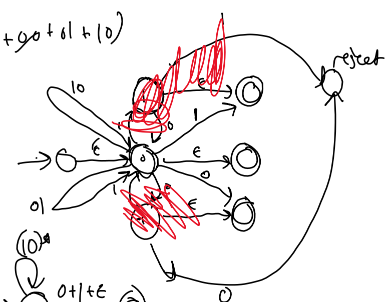
$\{ w \in \Sigma^* \mid \forall x \in \Sigma^*, s \in \Sigma^*, \text{ s.t. } w = xs, \; |\#_0(x) - \#_1(x)| \leq 1 \}$

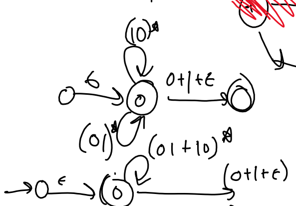


how many more 1's than 0's

$(10 + 01)^* (1 + 0 + \epsilon)$

$(\cancel{11} + \cancel{00} + 01 + 10)$

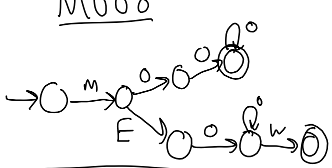




$(01 + 10)^* (0 + 1 + \epsilon)$

$\boxed{20}$ $\underline{M}\cancel{O}(O^*) + \underline{M}EO(O^*)W$

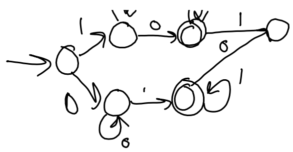
$\underline{MOOO^*}$



$\boxed{24}$ "Strings containing $\underline{10}$ or $\underline{01}$ but $\underline{\text{not}}$ both"

$\rightarrow (1^*) 10 (0^*) +$

$\rightarrow (0^*) 01 (1^*)$



$\boxed{3S.}$ $\{ 0^{F_n} \mid n \geq 0 \}$ "Prove regular or not"

where $F_n = \begin{cases} 0 & n=0 \\ 1 & n=1 \\ F_{n-2} + F_{n-1} & n \geq 2 \end{cases}$

Prove L has an infinite fooling set $S = \{0^n \mid n \in N\}$

$S = \{ 0^{F_n} \mid n > 10 \}$ Inf size ✓ choose some large enough to avoid $\min(i,j) < 2$

Let $x = 0^{F_i}$ and $y = 0^{F_j}$ be elements from S.

Hint: If $F_i + F_j$ is a Fib number then $|i-j| \leq 1$ or $\min(i,j) \leq 2$

Say F_i is the smaller one.

Let $z = 0^{F_{i-1}}$ ... $0^{F_i + F_{i-1}} = 0^{F_{i+1}}$ ✓

$0^{F_j + F_{i-1}}$ is not a fib.

Assert $|i-j| > 1$ ✓ $\min(i,j) > 2$. $j - (i-1) > 2$. because we chose 10

Therefore $F_i + F_j$ is NOT a Fib number.

$xz \in L$, $yz \notin L$. So S is an inf fooling set. ✓

$\boxed{4S}$ Is $L = \{ www \mid w \in \Sigma^* \}$ regular?

$L \neq (\Sigma^*)(\Sigma^*)(\Sigma^*)$ $a^n b^n$

$S = \{ w \in \Sigma^* \}$,

Let $x, y \in S$ $x \neq y$ $x = 000$ $y = \epsilon$ xxx 000 000 000 000 000

Let $z = xx$. $xz = xxx \in L$.

$yz = yxx \in L$

$S = \{ 0^n 1 \mid n \in N \}$

Let $x = 0^i 1$, $y = 0^j 1$, $i \neq j$ Let i be $i < j$.

$z = xx$. $xz = 0^i 1 0^i 1 0^i \in L$

$yz = 0^j 1 0^i 1 0^i 1$

if $|yz| = 3|x|$, and $yz = www$.

then $w \notin 0^*$...

$\boxed{3}$ Evens: evens(w) is the even number subsequence.

e.g. evens$(0\underline{1}1\underline{0}1\underline{0}) \rightarrow 100$

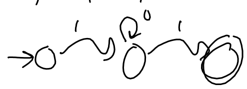
$0\underline{1}0\underline{1}0 \rightarrow 11$

Let L be a reg lang.

$\underline{L'} = \{ \underline{evens(w)} \mid \underline{w \in L} \}$ is regular.

$= \{ w \mid w = evens(x) \text{ for some } x \in L \}$

Let $M = (Q, \delta, S, A)$ be a DFA for L.

$M =$  $\underline{1}0\underline{0}1$

$N =$  $Q' = \{ (q, b) \mid q \in Q, b \in \{E, O\} \}$

$\delta'((\underline{q, O}), \underline{\epsilon}) = \{ (\delta(q, \underline{1}), \underline{E}), (\delta(q, \underline{0}), E) \}$

$\delta'((q, E), \underline{a}) = \{ (\delta(q, a), O) \}$

$S' = (S, O)$ $\quad A' = \{ (q, b) \mid q \in A \}$

$\boxed{10.3}$ Prove this grammar accepts

$L = \{ 0^m 1^n \mid \underline{n \leq 2m \text{ and } m \leq 2n} \}$

$S \to A \mid B$

$A \to \underline{00A1} \mid \underline{C}$

$B \to 0B11 \mid C$

$C \to \underline{0}C\underline{1} \mid \epsilon$

Let $0^m 1^n$ be a string in L.

If $m = n$ $\quad S \leadsto A \leadsto C \leadsto^m 0^m 1^m$ ✓

$m > n$
$m \geq n+1$ $\quad S \leadsto A \leadsto 00A1$

$A \leadsto^* 0^{m-2} 1^{n-1}$

Apply A $m-n$, C $(2n-m)$ $\quad S \leadsto^* 0^{2a+c} 1^{a+c}$

$0^{2(m-n)+(2n-m)} \; 1^{(m-n)+(2n-m)}$

So if $m > n$,

$= 0^{2m-2n+2n-m} \; 1^{m-n+2n-m}$

$0^m 1^n$

Let $m \geq n+1$.

IH: If $(m'-n') < m-n$, then $A \leadsto 0^{m'} 1^{n'}$.

$A \leadsto 00A1$. $\quad 0^m 1^n = 00\, 0^{m-2} 1^{n-1} 1$

$m' = m-2 \quad n' = n-1$ $\quad m-2 \; n-1$

Let

$m'-n' < m-n$. So by IH, $A \rightsquigarrow 0 \; 1$.

$C \rightsquigarrow^{*} \{0^n 1^m\}$