Today

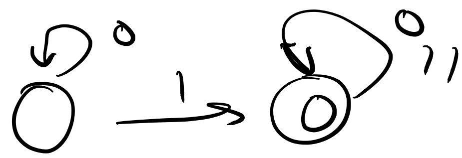
- Myhill-Nerode
- Non-reg via closure
- CFG
  - definition
  - examples
  - properties
  - parse trees

Def'n Given a language L, F is a fooling set if for any distinct $x, y \in F$ $\exists w$ such that $xw \in L$ and $yw \notin L$ or vice versa.

Thm (Myhill-Nerode) Given a language L

1. If L has a fooling set F with $|F| = n$ then no DFA w/ $< n$ states accepts L
2. If L has an infinite fooling set, then L is not regular $\equiv$ no DFA accepts L,
3. If L has a maximal fooling set F with $|F| = n$, then there is a DFA w/ n states that accepts L

$L_1 = (0+1)^* 1 (0+1)^*$



$F_1 = \{0, 1\}$

$L = 0^n 1^n$ $\quad F = \{0, 00, 000, 0000\}$

$L_2 =$ binary strings divisible by 3

$\{11, 110, 1001, 1100, \ldots\}$

$F_2 = \{11, 10, 1\}$, 101

$\underset{x}{10}$ $\underset{w}{1}$ $\quad$ $\underset{y}{1}$ $\underset{w}{1}$

$L_3 = \Sigma^*$ $\quad$ ~~$F_3 = \emptyset$~~ $\quad$ $F_3 = \{\epsilon\}$

Suppose L_1, L_2, L_3

$L_3 = L_1 \cap L_2$

L_2 is regular

L_3 is not regular

L_1 is not regular

$L_3 = \{0^n 1^n \mid n \geq 0\}$ is not regular

$L_1 = \{w \in \{0,1\}^* \mid \#0(w) = \#1(w)\}$

$L_3 = L_1 \cap 0^* 1^*$

$L_4 = \Sigma^* - 0^n 1^n$ is not regular

L_5 is regular L_6 is not regular

$L_7 = L_5 \cap L_6$

$L_6 = 0^n 1^n$ $L_5 = \emptyset$

$L_7 = \emptyset$

$L_8 = L_5 \cup L_6$ $L_7 = \Sigma^*$

$L_5 = \Sigma^*$

L_9, L_{10} are not regular

$L_{11} = L_9 \cup L_{10}$ if e.g. $L_9 = L_{10}$ L_{11} not reg

OTOH $L_9 = 0^n 1^n$

$L_{10} = 0^n 1^m \quad n \neq m$

$L_9 \cup L_{10} = 0^* 1^*$

Sum up

show a lang not regular

- infinite fooling set
- (careful) closure argument
- pumping lemma

All languages

RL - finite memory
- linear time
- encode repeated patterns

CFL - infinite, stack-based memory
- polynomial time
- encode recursion

$r = \emptyset \qquad r = \varepsilon \qquad r = c \in \Sigma$

$r = r_1 \cdot r_2 \qquad r = r_1 + r_2 \qquad r = r_1^*$

$R \to \emptyset \mid \varepsilon \mid c \mid R \cdot R \mid R + R \mid R^*$
$\mid (R)$

$E \to E + E \mid E - E \mid E * E \mid N$

$N \to 1 \mid 2 \mid 3 \mid \ldots$
$N1 \mid N2 \mid N3$

<program> → <stmts> | <stmt><program>

<stmts> → <assign> | <fun-def> | ..

<assign> → <var> '=' <expr>

...

<var> → a | b | c ...

CFG = (V, T, P, S)

V → variables, non-terminals

T → terminals, aka symbols aka alphabet

P → productions

$A \to b\,C$

non-terminal terminals + non-terminals

$P \subset V \times (V \cup T)^*$

(finite subsets)

$S \to$ starting non-terminal $\qquad S \in V$

CFG $\quad$ DFA

$T \equiv \Sigma$

$V \sim Q$

$P \sim \delta$

$S \sim s$

$? \sim A$

$T = \{a, b\} \qquad V = \{S\}$

$P = S \to a \mid b \mid aSa \mid bSb$

$P = \{S \to a, S \to b, \underline{S \to aSa}, S \to bSb\}$

Derivation

$S \rightsquigarrow a\,S\,a \rightsquigarrow a\,b\,S\,b\,a$

$\rightsquigarrow ab\,b\,ba \qquad S \rightsquigarrow^* abbba$

Formally $\quad G = (V, T, P, S)$

$\alpha_1, \alpha_2 \in (V \cup T)^* \qquad \alpha_1$ derives α_2

$\alpha_1 \rightsquigarrow_G \alpha_2$

if $\exists\ A \in V,\ \beta, \gamma, \delta \in (V \cup T)^*$

$\alpha_1 = \beta A \gamma \qquad A \to \delta \in P$

$\alpha_2 = \beta \delta \gamma$

$\alpha_1 \rightsquigarrow^k \alpha_2$

$=$

$\alpha_1 \rightsquigarrow \alpha_3 \rightsquigarrow^{k-1} \alpha_2$

$\alpha_1 \rightsquigarrow^0 \alpha_1$

$\underbrace{\alpha_1 \rightsquigarrow \_ \rightsquigarrow \_ \rightsquigarrow}_{k} \alpha_2$

$\alpha_1 \rightsquigarrow^* \alpha_2$

if $\alpha_1 \rightsquigarrow^k \alpha_2$ for some k

Given $G = (V, T, P, S)$

$L(G) = \{ w \in T^* \mid S \rightsquigarrow^* w \}$

$T = \{a, b\}$ $\quad V = \{S\}$

$P = S \to a \mid b \mid aSa \mid bSb$

$L(G) =$ palindromes odd length

$S \to a \mid b \mid aSa \mid bSb \mid \epsilon$

pal of any length

$L = 0^n 1^n$

$S \to 01 \mid \epsilon \mid 0S1$ $\quad 000$

$L = 0^0 1^0$

$S \to \epsilon \mid 01 \mid 0S1$

$0S \mid S1$

$L_1 = 0^n 1^m$ $\quad m > n$

$= 0^n 1^n 1^k$ $\quad n \geq 0, k \geq 1$

$S_1 \to 1 \mid 0S_11 \mid S_11$

$L_3 = 0^n 1^m \quad n \neq m$

$= 0^n 1^m \quad m > n \text{ OR } m < n$

$L_2 = 0^n 1^m \quad m < n$

$S_2 \to 0 \mid 0 S_2 1 \mid 0 S_2$

$S_3 = S_1 \mid S_2$

$G_1 = (V_1, T, P_1, S_1) \quad V_1 \cap V_2 = \emptyset$

$G_2 = (V_2, T, P_2, S_2)$

$G_3 \quad L(G_3) = L(G_1) \cup L(G_2)$

$V_3 = V_1 \cup V_2 \cup \{S_3\}$

$P_3 = P_1 \cup P_2 \cup \{S_3 \to S_1 \mid S_2\}$

$G_4 \quad L(G_4) = L(G_1) \cdot L(G_2)$

$S_4 \to S_1 S_2$

$G_5 \quad L(G_5) = L(G_1)^*$

$S_5 \to \epsilon \mid S_1 S_5$