Reg Langs & DFA
Tuesday, January 28, 2020 11:49 AM

Regular expressions

$R = \varepsilon$
$\mid [a]$
$\mid R_1 R_2$
$\mid R_1 + R_2 \overset{\text{means}}{\Rightarrow} r(R_1 + R_2) = r(R_1) \cup r(R_2)$ $R_1 \cup R_2$
$\mid R_1^* \Rightarrow r(R)^* = \bigcup_{n \in N} \{R^n\}$

ex. $\underline{R_{21} = (0^* 1 0^* 1 0^*)}$
"all strings containing two 1's"?
11 ∈ R
0's can go any of three places

ex. "all string with even # of 1's"

$(R_{21})^*$?

$0^* + (R_{21})^+$ ✓

- Reg. Lang closed under ∪ ✓
- Reg. Lang closed under ∩ motivating q, for later
- Reg. Lang closed under Complement

$\overline{R} = \{w \mid w \in \Sigma^*, w \notin R\}$

Strings contain "0 1 1" $(0+1)^* 011 (0+1)^*$

- ex. $\{0^n 1^n \mid n \in N\}$ is not regular

3 0's and 3 1's

$(0^1 1^1 + 0^2 1^2)^*$

01 00 11

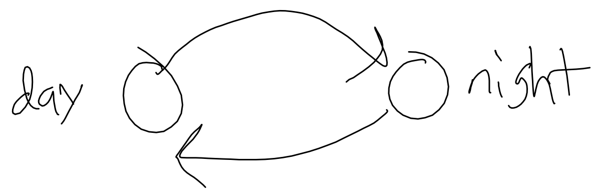
DFA and State machines

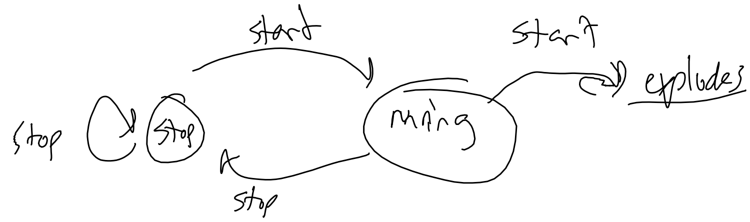
Deterministic Finite State Automata

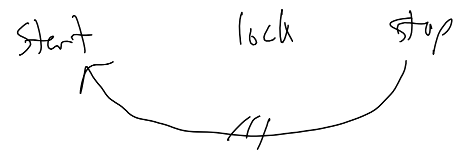
⇓

equiv. to regular expressions

State



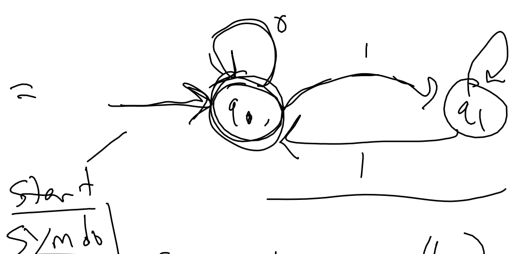


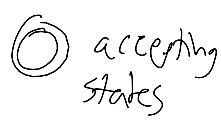


Students in 374

State machine for recognizing $\Sigma = \{0, 1\}$

M =





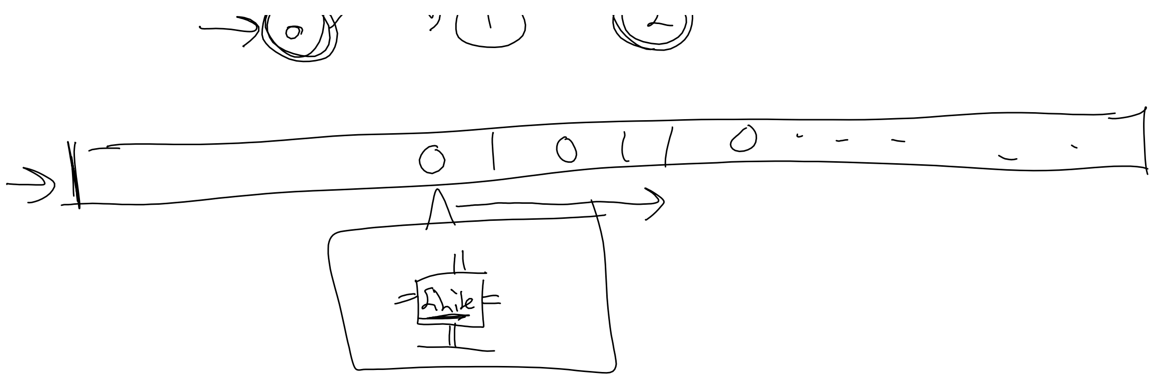
walk(M, "0110") ✓

q_0 q_0 q_1 q_0 q_0

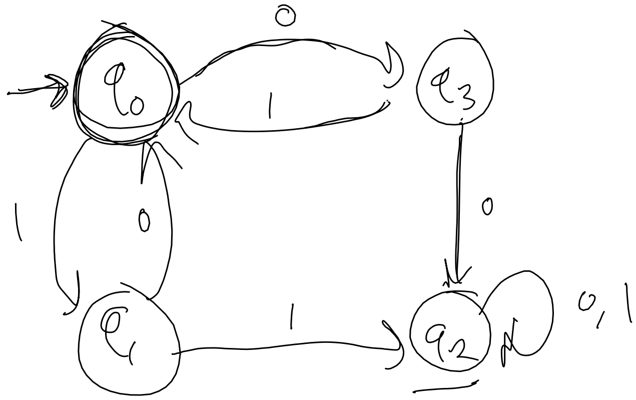
$x \in L(M)$
iff walk(M, x) ∈ AcceptStates

"any string with even # of 1s"

n_ones = 0
for i in |x|:
n_ones ++ if x[i] = 1
return 1 if n_ones % 2 = 0



Ex.



0 0 1

$q_0\ q_3\ q_2\ q_2$

"Can't have consecutive chars" X

$((01)+(10))^*$ ✓ 01 10 01 01

1001 ✓

"all binary strings that when treated as integers (big endian), that are multiples of 5"

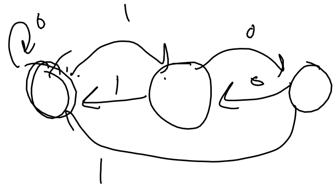
$\underset{8\ 0\ 2\ 1}{1011} = 11 = 1 \bmod 5$ X

$1010 = 10 = 0 \bmod 5$ ✓

$101 = 5 = 0 \bmod 5$ ✓

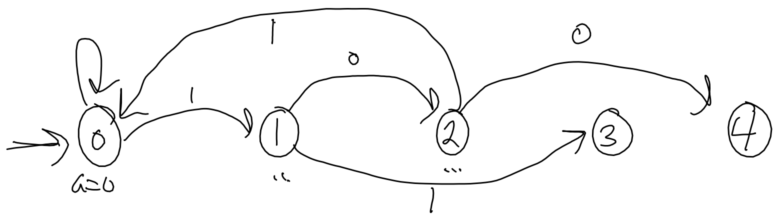
Reg exp?

$1 \to 0$ & get to 00



?

$11 = 3$ X



observation:

$w = a \bmod 5$

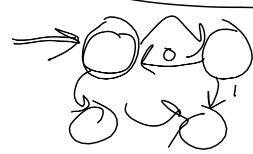
$w0 = 2a \bmod 5$

$w1 = 2a+1 \bmod 5$

$101 = 5$

$1011 = 11$

Formal Def'n of DFA



$$M = (\underbrace{Q}_{\text{states}}, \underbrace{\Sigma}_{\text{alphabet}}, \underbrace{\delta}_{\text{transition function}}, \underbrace{s}_{\text{start state}}, A)$$

start ~~symbol~~ state

Q is a finite set $\quad |Q| \in \mathbb{N}$

$|\Sigma| \in \mathbb{N}$

$\delta : (Q \times \Sigma) \to Q$

$\delta : (Q \times \Sigma) \to Q$

$s \in Q$

$A \subseteq Q$

$walk(M, w) = \delta \ldots \delta(\delta(s, w[0]), w[1]) \ldots)$

~~if |w| = 0, then s~~

~~(a, x), then~~

walk'(M, w, q) :

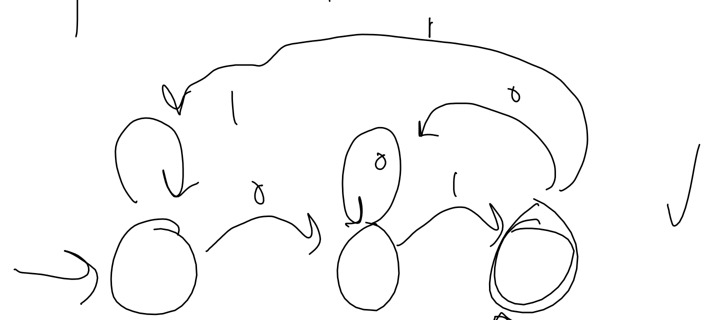
if |w| = 0 then q

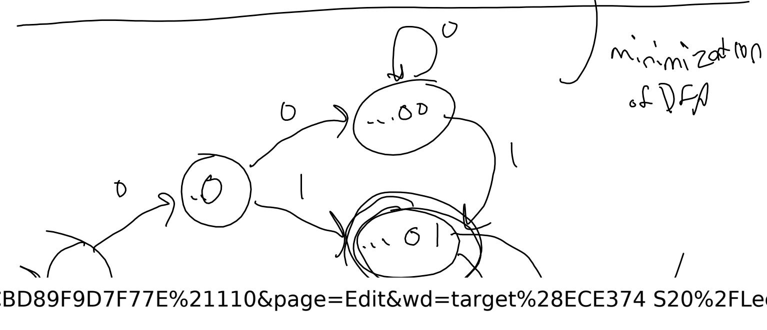
else w=ax then walk'(M, x, δ(q, a))

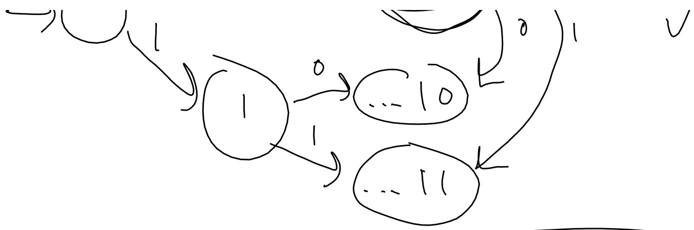
walk(M, w) = walk'(M, w, s)

note: walk(M, w) is $\delta^*(w, s)$ in slides

$L = \{ w \mid w \in \Sigma^*, w \text{ ends in } \text{"}\ldots 01\text{"} \}$







$\delta^* : Q \times \Sigma^* \to Q$

$\delta^*(q, w) =$

$\delta^*(q, \varepsilon) = q$

$\delta^*(q, a \cdot x) = \delta^*(\delta(q,a), x)$

$L(M) = \{ w \mid w \in \Sigma^* \ \ \delta^*(s, w) \in A \}$

$\forall$ strings x, symbols a, and DFAs $M = (Q, \delta, s, A, \Sigma)$

Thm: $\underline{\delta^*(q, x \cdot \underline{a})} = \underline{\delta(\delta^*(q, x), a)}$

Proof: By induction on X

Base case: $x = \varepsilon$ $\quad |x| = 0$

Goal: $\delta^*(q, a) = \delta(\underbrace{\delta^*(q, \varepsilon)}_{\parallel}, a)$

$= \delta(q, a)$

✓

by base case of Def'n of δ^*

Inductive Case: $|x| = n+1$ $\quad |y| = n$

$x = by$

IH: $\forall q'\!: \ \delta^*(q', y \cdot a) = \delta(\delta^*(q', y), a)$

Goal $\delta^*(q, \underset{b(ya)}{b \cdot y a}) = \underline{\delta(\delta^*(q, b \cdot y), a)}$

$\delta^*(\delta(q, b), ya)$

Let $q' = \delta(q, b)$

By IH $\delta^*(\delta(q,b), ya) = \delta(\delta^*(\delta(q,b),y),a)$

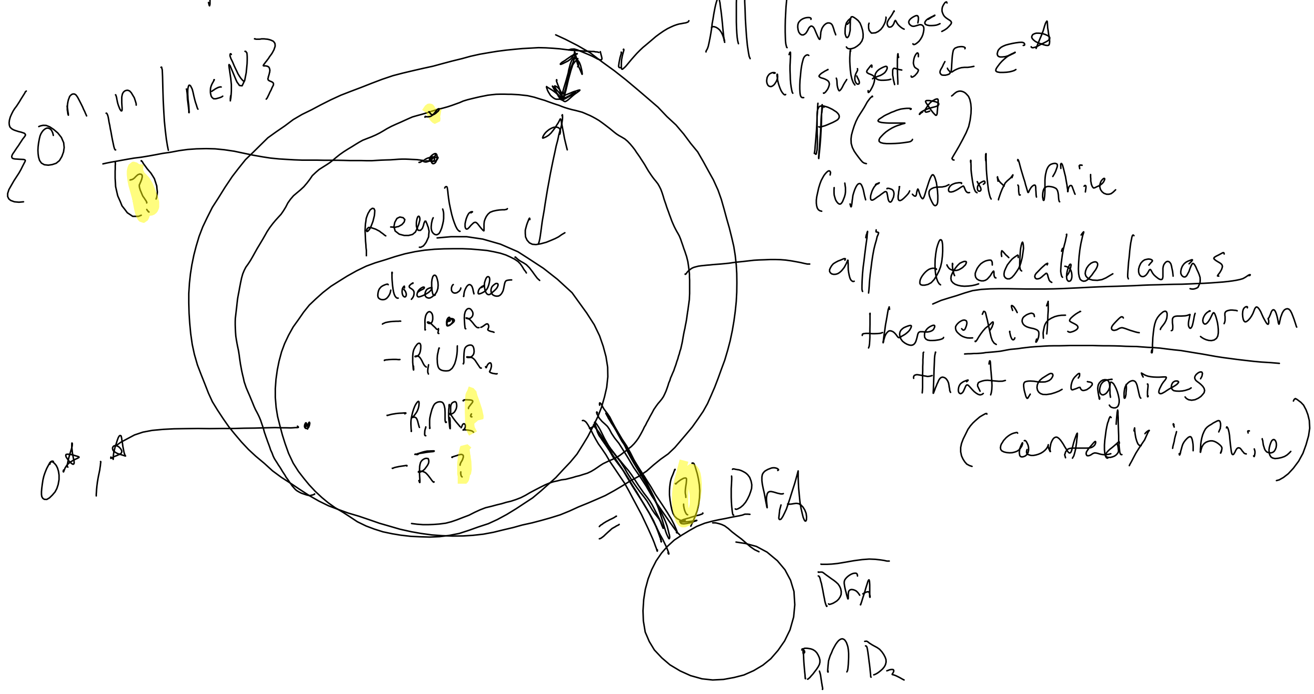
$= \delta(\delta^*(q,by),a)$ by defn δ^*

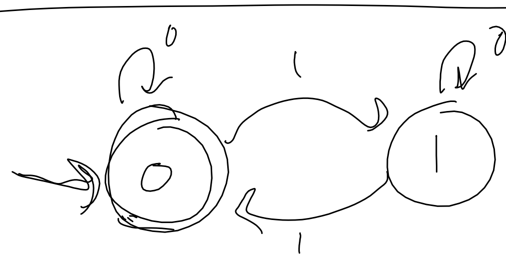
✓

Thm: Let M = 

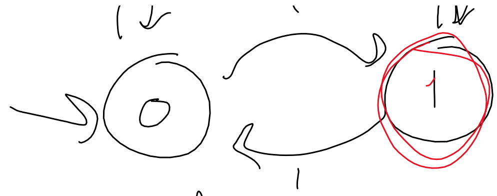
$L(M) = \{ w \mid \#_1(w) \text{ is even} \}$

Product Construction



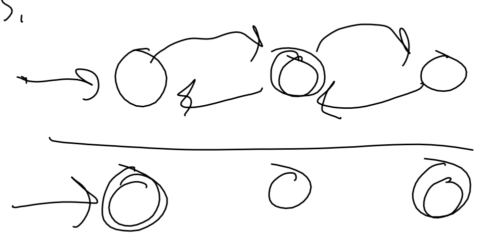


even



odd

3 nodes?



$$M = (\Sigma, Q, \delta, s, A)$$

$$\overline{M} = (\Sigma, Q, \delta, s, \bar{A} = Q - A)$$

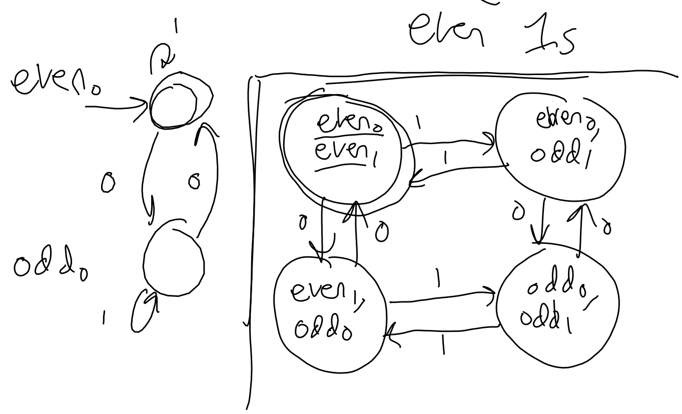
$$\overline{L(M)} = L(\overline{M})$$

complement ✓

Product Construction

even 1s

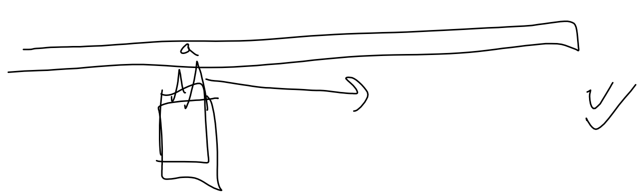
$\{ w \mid \#_1(w) = \text{even}$ and $\#_0(w) = \text{even} \}$



$\cap$

$\{ \dots \#_1(w) \text{ even} \}$

$\{ \dots \#_0(w) \text{ even} \}$



✓

$$M_1 = (Q_1, \Sigma, \delta_1, s_1, A_1)$$

$$M_2 = (Q_2, \Sigma, \delta_2, s_2, A_2)$$

$$M_{1\times 2} = (Q_1 \times Q_2, \Sigma, \delta_{1\times 2}, (s_1, s_2), A_{1\times 2})$$

$$(s_1, s_2) \in Q_1 \times Q_2$$

$$A_{1\times 2} = A_1 \times A_2 = \left\{ (q_1, q_2) \in Q_1 \times Q_2 \;\middle|\; \begin{matrix} q_1 \in A_1 \\ q_2 \in A_2 \end{matrix} \right\}$$

$$\delta_{1\times 2} : (Q_1 \times Q_2) \times \Sigma \rightarrow (Q_1 \times Q_2)$$

$$\delta(\underline{(q_1, q_2)}, \underline{a}) = \underline{(\underline{\delta_1}(q_1, \underline{a}), \underline{\delta_2}(q_2, \underline{a}))}$$