Reg Langs & DFA
Tuesday, January 28, 2020 11:49 AM

Regular expressions

$R = \varepsilon$
$\mid [a]$
$\mid R_1 R_2$
$\mid R_1 + R_2$ means $\Rightarrow r(R_1 + R_2) = r(R_1) \cup r(R_2)$ $\quad R_1 \cup R_2$
$\mid R_1^*$ $\Rightarrow r(R)^* = \bigcup_{n \in \mathbb{N}} \{R^n\}$

ex. $R_{21} = (0^* 1 0^* 1 0^*)$

"all strings containing two 1's"?

$\Rightarrow \in R$

0's can go any of three places

ex. "all string with even # of 1's"

$(R_{21})^*$?

$0^* + (R_{21})^+$ ✓

- Reg. Lang closed under $\cup$ ✓
- Reg. Lang closed under $\cap$ motivating q, for later
- Reg. Lang closed under complement

$\overline{R} = \{w \mid w \in \Sigma^*, w \notin R\}$

strings with "0 1 1" $\quad (0+1)^* 011 (0+1)^*$

- ex. $\{0^n 1^n \mid n \in \mathbb{N}\}$ is not regular

3 0's and 3 1's

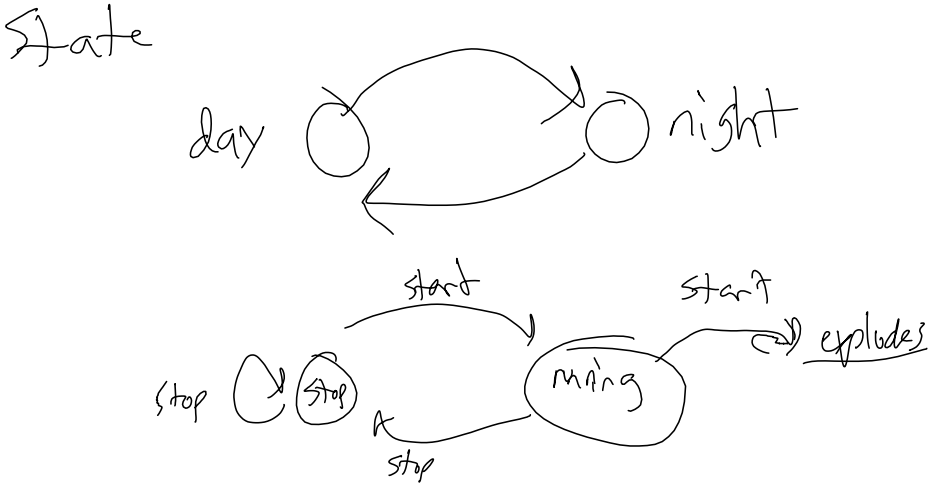
$(0^1 1^1 + 0^2 1^2)^*$

0 1 00 11

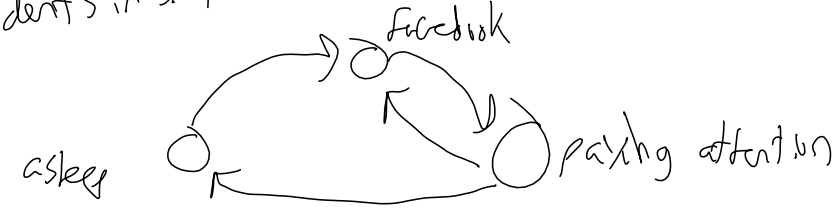
DFA and State machines

Deterministic finite State Automata

equiv. to regular expressions

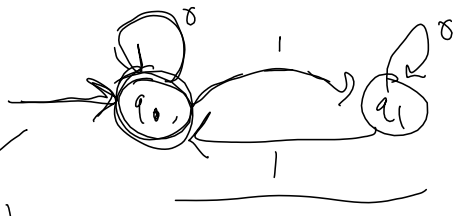


Students in 374



State machine for recognizing $\Sigma = \{0, 1\}$

$M =$



start symbol

accepting states

walk(M, "0110") → $q_0\ q_0\ q_1\ q_0\ q_0$

$X \in L(M)$

iff walk(M, X) $\in$ AcceptStates

"any string with even # of 1s"

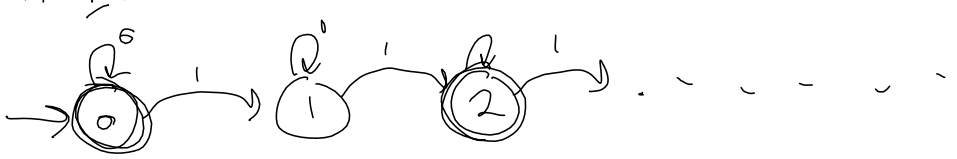
~~num ones = 0~~

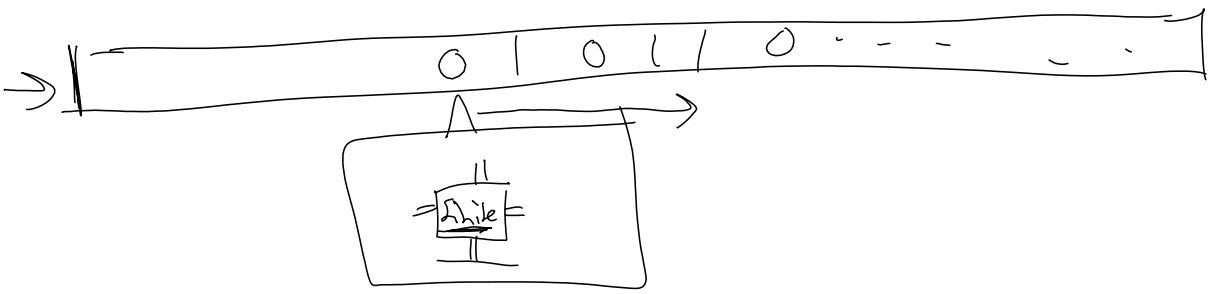
for i in |X|:

if x[i] = 1

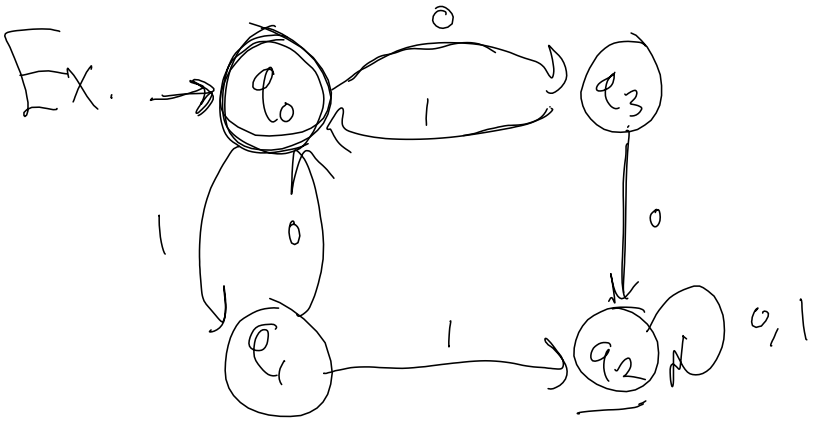
n_ones ++ if 1

return 1 if n_ones % 2 = 0





Ex.



0 0 1

q_0 q_3 q_2 q_2

"Can't have consecutive chars" X

$((01)+(10))^*$ ✓

01 10 01 01

1 0 0 1 ✓

"all" binary strings that when treated as integers (big endian), that are multiples of 5"

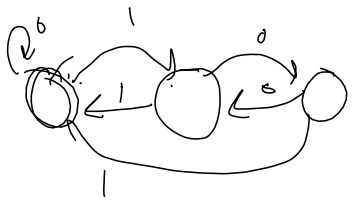
$\underbrace{1011}_{8\ 0\ 2\ 1} = 11 = 1 \bmod 5$ X

$|10|0 = |0| = 0 \bmod 5$ ✓

$|1|1 = 5 = 0 \bmod 5$ ✓

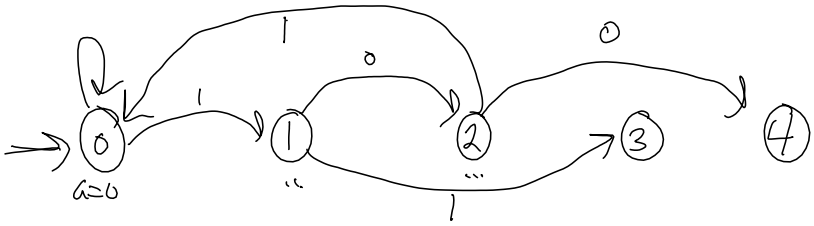
Reg exp?

$1 \to 0$ & get to q_0



?

$|1| = 3$ X



Observation:

$w = a \bmod 5$

$w0 = 2a \bmod 5$

$w1 = 2a+1 \bmod 5$

$|01| = 5$

$|011| = 11$

Formal Defn of DFA

$M = (Q, \Sigma, \delta, s, A)$

Q: states

Σ: alphabet

δ: transition

s: start symbol

function

Q is a finite set $|Q| \in \mathbb{N}$

$|\Sigma| \in \mathbb{N}$

$\delta : (Q \times \Sigma) \to Q$

$s \in Q$

$A \subseteq Q$

$walk(M, w) = \delta \ldots \delta(\delta(s, w[0]), w[1]) \ldots)$

~~if $|w| = 0$, then s~~

~~(a, X), then~~

walk'(M, w, q):

if $|w| = 0$ then q

else $w = ax$ then walk'(M, x, $\delta(q, a)$)

walk(M, w) = walk'(M, w, s)

note: walk(M, w) is $\delta^*(w, s)$ in slides