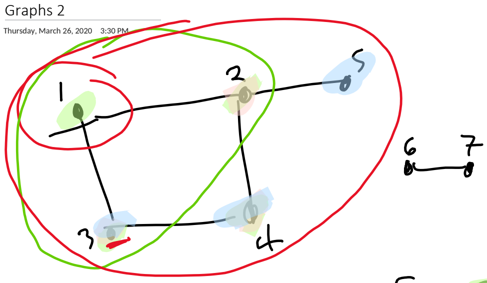
Graphs 2

Thursday, March 26, 2020 3:30 PM



DFS Stack: [1 3 ~~2~~ ~~8~~ ~~4~~ ~~5~~ ← access from last

BFS Queue: → [4]
access from first

Explore(1

Problem: Count or label the connected components.

DFS(G): $G=(V,E)$
- for $n \in V$:
  - if n not visited:
    - Explore(n)

Explore(n):
- if n not visited:
  - for each $n' \in$ neighbors of n:
    - mark n' as visited:
    - Explore(n')

Tools at disposal:

→ Traverse a graph

— Store an auxiliary array.
ex. visited[n]

— Build an auxiliary graph.

— optionally: associate labels to nodes or edges
G.nodes[n]["comp no"] := ctr
component number

Finding cycles, topological order

Categories:

— Directed graphs

— Directed Acyclic Graphs (DAG):
Contains no cycle

$(1,2) \in E$

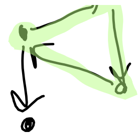
$(2,1) \notin E$

— Forest: each node has only one incoming edge

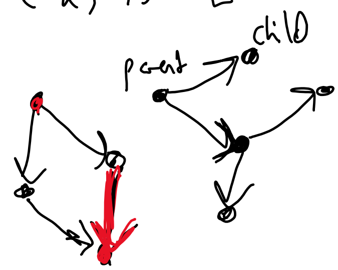
— Tree: has one root

Defn: Cycle: $v_1 \to v_2 \to v_3 \to \dots \to v_k$

s.t. $\forall\, v_i, v_{i+1}$, $(v_i, v_{i+1}) \in E$

ex.  and $(v_k, v_1) \in E$

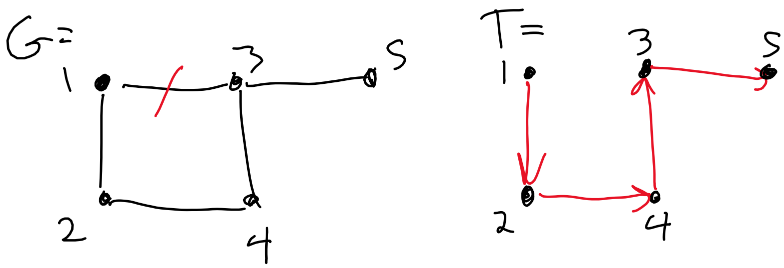
ex. Forest.



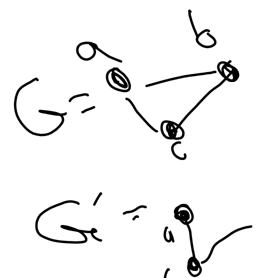
Claim: DFS(G) for any graph defines a spanning forest

Def'n: Spanning tree/forest: is a subgraph containing all the nodes. and that is a tree/forest

Def'n: $G' = (V', E')$ is a subgraph of $G = (V, E)$ iff $V' \subseteq V$ $E' \subseteq E$ and G' is a graph.

ex. G= 

T is a spanning tree of G

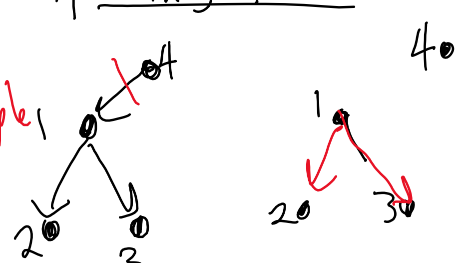
G= 

$= (\{a, c\}, \{(a,c), (c,b)\})$

E' must $\subseteq V'^2$

Q: Does DFS(G) for a directed tree G, give a spanning tree?

No, counterexample,

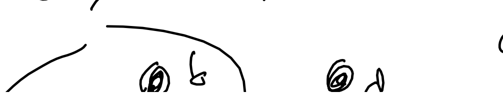


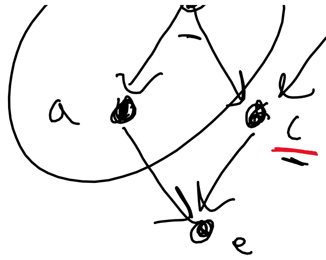
Topological order for a directed graph is an ordering $n_1 \prec n_2$ defined on V

↑ bendy less-than

s.t. If $(n_1, n_2) \in E$ then $n_1 \prec n_2$

$a \prec b \prec c \prec d \prec e$





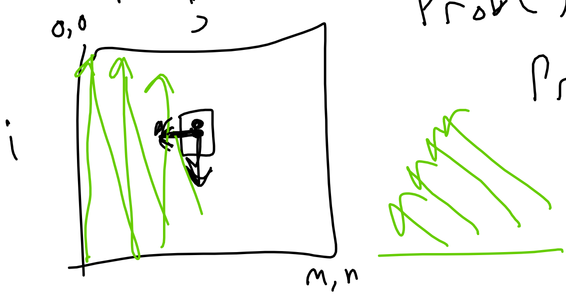
a b c d e

$(b,a) \in E$ yet $b \not< a$

X no this is not a top. ordering.

b, d, a, c, e also b a d c e

e.x. dyn prog



Prob(i,j) depends on Prob(i',j')

Graph:
nodes: subproblems (i,j)
edges: dependencies prob(i,j) depends on (i, j-1) and (i+1, j)

Pre-visit order, post-visit order:
please see slides [DFS]