1 Suppose you are given a magic black box that somehow answers the following decision problem in *polynomial time*:

- INPUT: A CNF formula φ with n variables $x_1, x_2, \ldots, x_n$.
- OUTPUT: TRUE if there is an assignment of TRUE or FALSE to each variable that satisfies φ.

Using this black box as a subroutine, describe an algorithm that solves the following related search problem *in polynomial time*:

- INPUT: A CNF formula φ with n variables $x_1, \ldots, x_n$.
- OUTPUT: A truth assignment to the variables that satisfies φ, or NONE if there is no satisfying assignment.

(**Hint:** You can use the magic box more than once.)

2 An ***independent set*** in a graph G is a subset S of the vertices of G, such that no two vertices in S are connected by an edge in G. Suppose you are given a magic black box that somehow answers the following decision problem *in polynomial time*:

- INPUT: An undirected graph G and an integer k.
- OUTPUT: TRUE if G has an independent set of size k, and FALSE otherwise.

2.A. Using this black box as a subroutine, describe algorithms that solves the following optimization problem *in polynomial time*:

- INPUT: An undirected graph G.
- OUTPUT: The size of the largest independent set in G.

(**Hint:** You have seen this problem before.)

2.B. Using this black box as a subroutine, describe algorithms that solves the following search problem *in polynomial time*:

- INPUT: An undirected graph G.
- OUTPUT: An independent set in G of maximum size.

To think about later:

3 Formally, a ***proper coloring*** of a graph $G = (V, E)$ is a function $c \colon V \to \{1, 2, \ldots, k\}$, for some integer k, such that $c(u) \neq c(v)$ for all $uv \in E$. Less formally, a valid coloring assigns each vertex of G a color, such that every edge in G has endpoints with different colors. The ***chromatic number*** of a graph is the minimum number of colors in a proper coloring of G.

Suppose you are given a magic black box that somehow answers the following decision problem *in polynomial time*:

- INPUT: An undirected graph G and an integer k.
- OUTPUT: TRUE if G has a proper coloring with k colors, and FALSE otherwise.

Using this black box as a subroutine, describe an algorithm that solves the following ***coloring problem*** *in polynomial time*:

- INPUT: An undirected graph G.
- OUTPUT: A valid coloring of G using the minimum possible number of colors.

(**Hint:** You can use the magic box more than once. The input to the magic box is a graph and **only** a graph, meaning **only** vertices and edges.)