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Prove that each of the following languages is *not* regular.

- 1  $\{0^{2^n} \mid n \geq 0\}$ .
- 2  $\{0^{2n}1^n \mid n \geq 0\}$
- 3  $\{0^m1^n \mid m \neq 2n\}$
- 4 Strings over  $\{0, 1\}$  where the number of 0s is exactly twice the number of 1s.
- 5 Strings of properly nested parentheses  $()$ , brackets  $[]$ , and braces  $\{\}$ . For example, the string  $([])\{\}$  is in this language, but the string  $([])$  is not, because the left and right delimiters don't match.
- 6 Strings of the form  $w_1\#w_2\#\dots\#w_n$  for some  $n \geq 2$ , where each substring  $w_i$  is a string in  $\{0, 1\}^*$ , and some pair of substrings  $w_i$  and  $w_j$  are equal.

### Extra problems

- 7  $\{0^{n^2} \mid n \geq 0\}$
- 8  $\{w \in (0+1)^* \mid w \text{ is the binary representation of a perfect square}\}$