F'18 Midterm Problem 3:

Given L,

$\text{prefix}(L) = \{ u \mid uv \in L \}$ $(u, v \in \Sigma^*)$.

To show: L is regular $\Rightarrow$ $\text{prefix}(L)$ is regular.

By regular expressions. (or DFAs / NFAs)

Same δ, $A' = \{ q \in Q :$ $\exists$ path from q to a state in A in DFA $\}$

$r = \emptyset: \Rightarrow \emptyset$

$r = \varepsilon \Rightarrow \varepsilon$

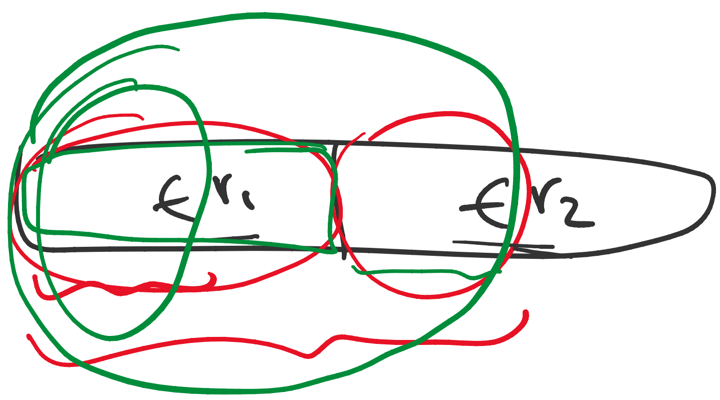
$r = a \ (a \in \Sigma) \Rightarrow a + \varepsilon$

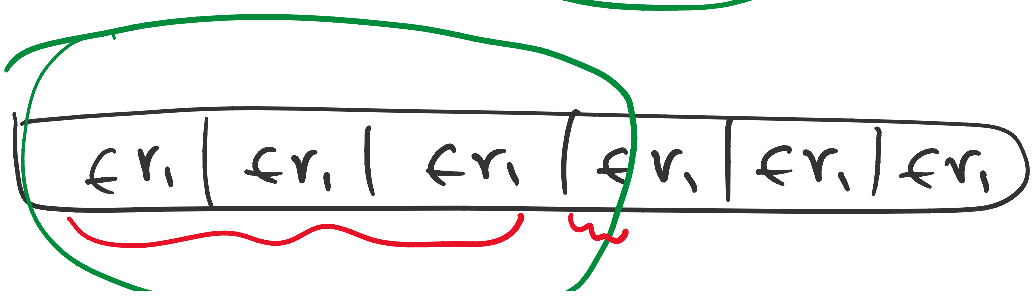
$r = r_1 + r_2:$ suppose s_1 for $\text{prefix}(L(r_1))$, s_2 for $\text{prefix}(L(r_2))$.

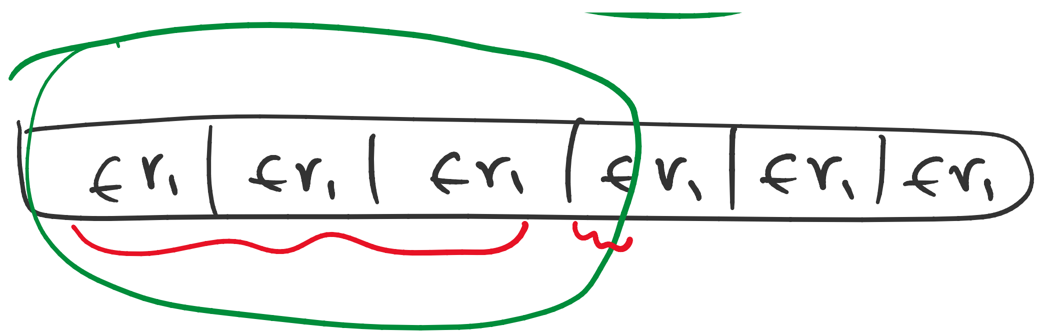
$\Rightarrow s_1 + s_2$

$r = r_1 r_2: \Rightarrow s_1 + r_1 s_2$

$r = r_1^*:$







$\Rightarrow \quad r_1^* s_1$

35. $\{ 0^{F_n} \mid n \geq 0 \}$ is not regular

F_n = Fibonacci numbers
($F_n = F_{n-1} + F_{n-2}$
$F_0 = 0$, $F_1 = 1$).

By fooling set method.

Choose $F = \{ 0^{F_n} \mid n \geq 2 \}$.

Let x, y be 2 arbitrary distinct strings from F.

Then $x = 0^{F_i}$ and $y = 0^{F_j}$ for some $i, j \geq 2$, $i \neq j$.

W.l.o.g. $i > j$. (otherwise, can swap $i < j$).

Choose $z = 0^{F_{i+1}}$

$\Rightarrow \quad xz = 0^{F_i + F_{i+1}} = 0^{F_{i+2}} \in L$

$\Rightarrow \quad xz = 0^{F_i + F_{i+1}} = 0^{F_{i+2}} \in L$

$\quad\;\; yz = 0^{F_j + F_{i+1}} \notin L.$

Since $F_j + F_{i+1}$ is not Fibonacci number.

$$F_{i+1} < F_j + F_{i+1} < F_i + F_{i+1} = F_{i+2}$$

ok

Thus, F is a fooling set.

Since F is infinite, L is not regular. $\square$

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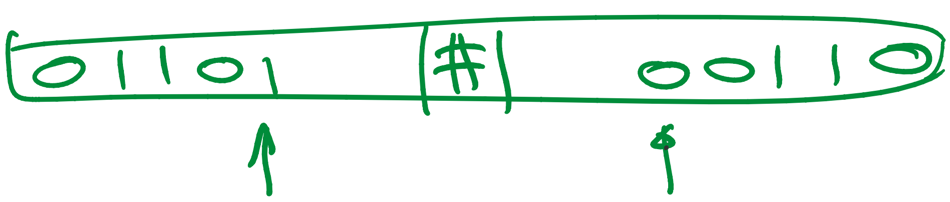
93. $\{ x \# y^R \mid x, y \in \{0,1\}^*, x \neq y \}$

Give CFG. $\Sigma = \{0, 1, \#\}$



$S \to 0S0 \mid 1S1 \mid 1A0 \mid 0A1 \mid 1B\#1 \mid 0B\#1 \mid \#B1 \mid \#B0$

11#1

$A \to 0A \mid 1A \mid \#B$

$A \to 0A \mid 1A \mid \#B$

$B \to 0B \mid 1B \mid \varepsilon$.

(A generates all strings with one #).

(~~Alternative:~~ ~~$A \to 0A \mid 1A \mid$~~ $\# \mid A0 \mid A1$)

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60. {all strings in $\{0,1\}^*$ s.t. # 0's is even
~~&~~ or # 1's is divisible by 3
~~&~~ or length is divisible by 5}.

Let $M = (Q, \Sigma, s, \delta, A)$.

$\Sigma = \{0,1\}$.

$Q = \{(i,j,k) : i \in \{0,1\}, j \in \{0,1,2\}, k \in \{0,1,2,3,4\}\}$

$= \{0,1\} \times \{0,1,2\} \times \{0,1,2,3,4\}$

$s = (0,0,0)$

~~$A = \{(0,0,0)\}$~~

$A = \{(i,j,k) : i=0 \text{ or } j=0 \text{ or } k=0\}$

$\delta((i,j,k), 0) = ((i+1) \bmod 2, j, (k+1) \bmod 5)$

$\delta((i,j,k), 1) = (i, (j+1) \bmod 3, (k+1) \bmod 5)$.

$(k+1) \bmod 5)$.

□

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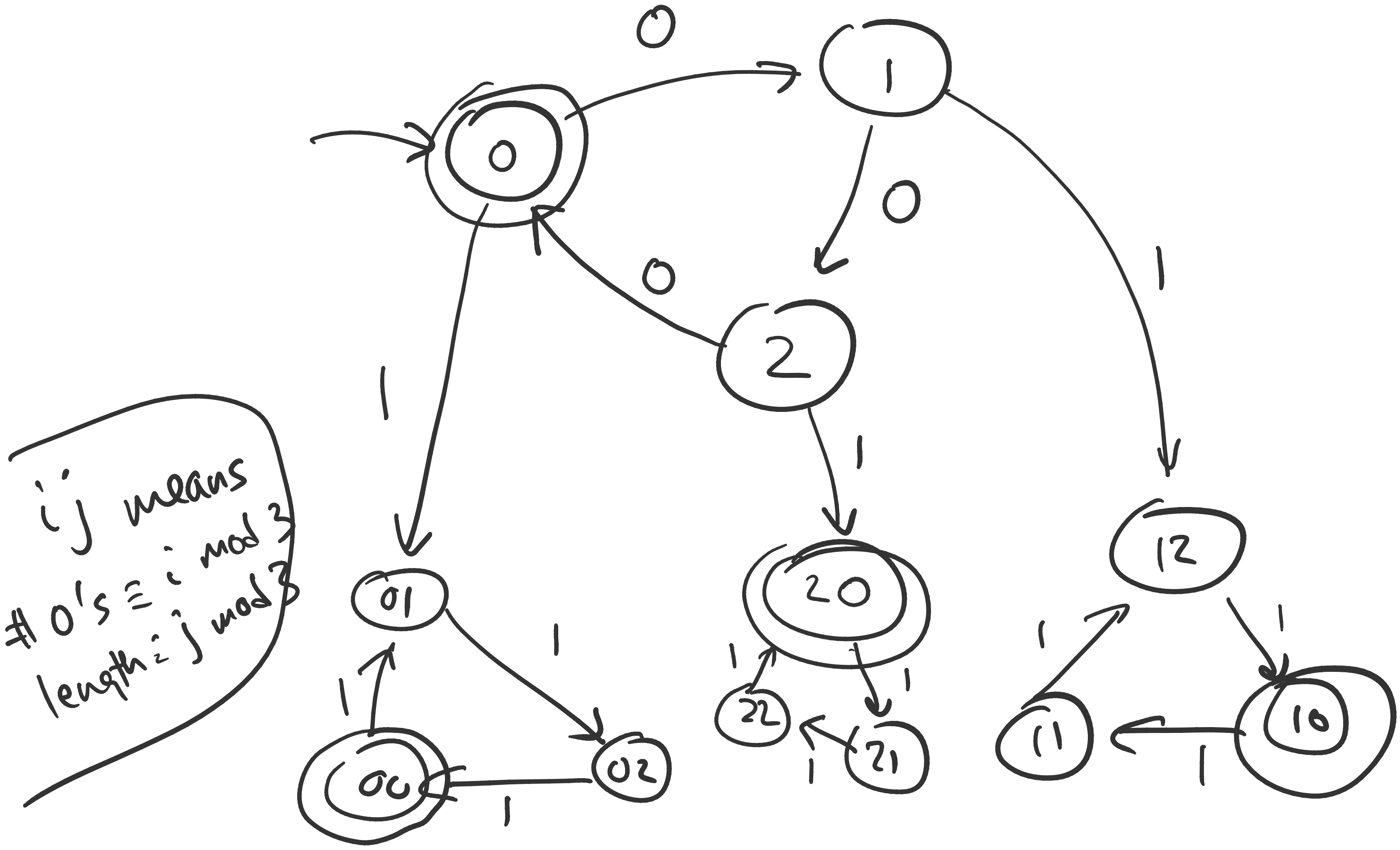
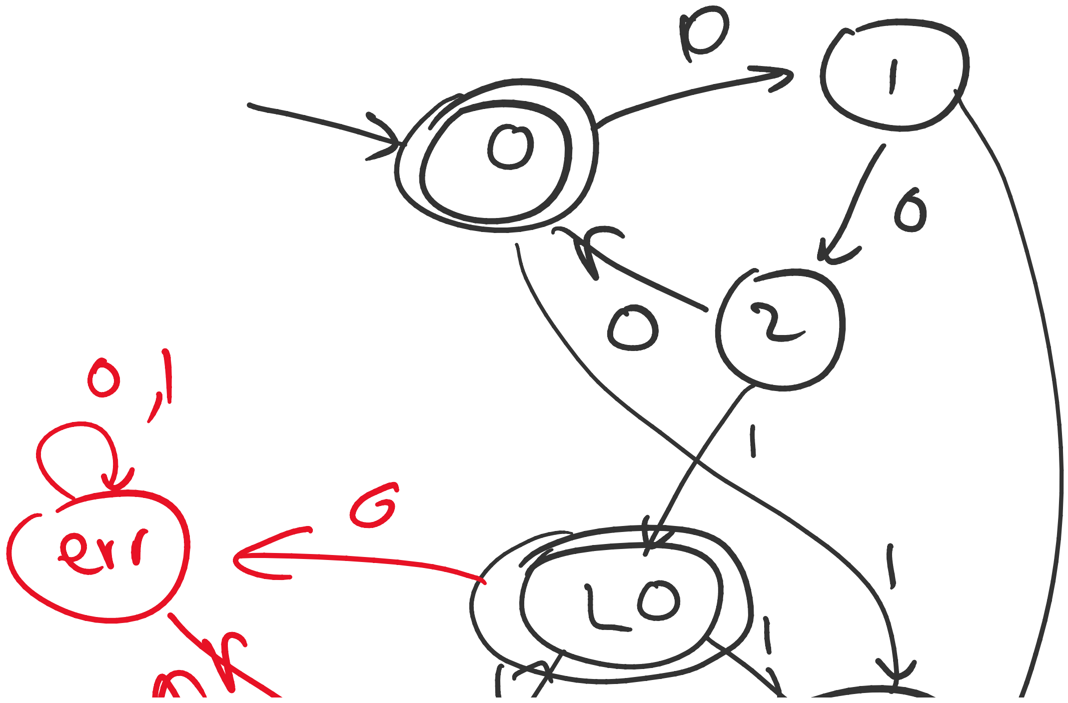
26. all strings in 0^*1^* with length div by 3

DFA:





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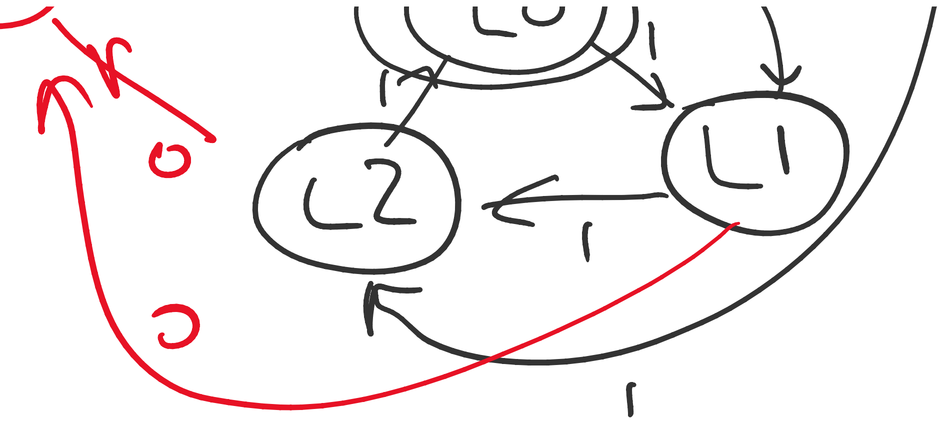

reg expr: $(000)^*(111)^*$

$+ \quad (000)^* 011 (111)^*$

$+ \quad 00(000)^* 1 (111)^*$

$(000)^* 0$
$\equiv 0(000)^*$

F'18 Midterm Problem 6:

Given CFGs $G_i = (V_i, T, S_i, P_i)$ $i=1,2,3$, for L_i, describe new CFG $G' = (V', T, S', P')$ for $L_1 + L_2 L_3^*$

$$P' = \{ S' \to S_1 \mid S_2 S'', \quad S'' \to S_3 S'' \mid \varepsilon \}$$

$$\cup \quad P_1 \cup P_2 \cup P_3.$$

$$V' = V_1 \cup V_2 \cup V_3 \cup \{S', S''\}$$

(disjoint unions)