Kleene's Thm ('51)

L is regular $\iff$ L is accepted by some DFA.

Corollary

If L is regular, then $\overline{L}$ is regular.

If L_1, L_2 are regular,
then $L_1 \cap L_2$ is regular.

If L is accepted by some DFA,
then so is L^R.

Proof Outline:

$$\begin{tikzcd} \text{regular} \arrow[r] & \text{NFA} \arrow[r] & \text{DFA} \arrow[ll, bend left] \end{tikzcd}$$

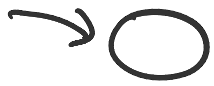
Regular → NFA

Lemma If L is regular,
then L is accepted by some NFA.

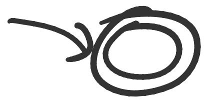
Pf Sketch:
method - recursion (i.e. induction)

Base cases.

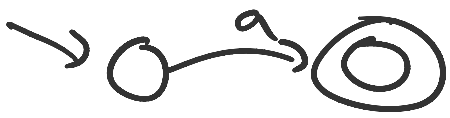
① ϕ



② ε



③ a

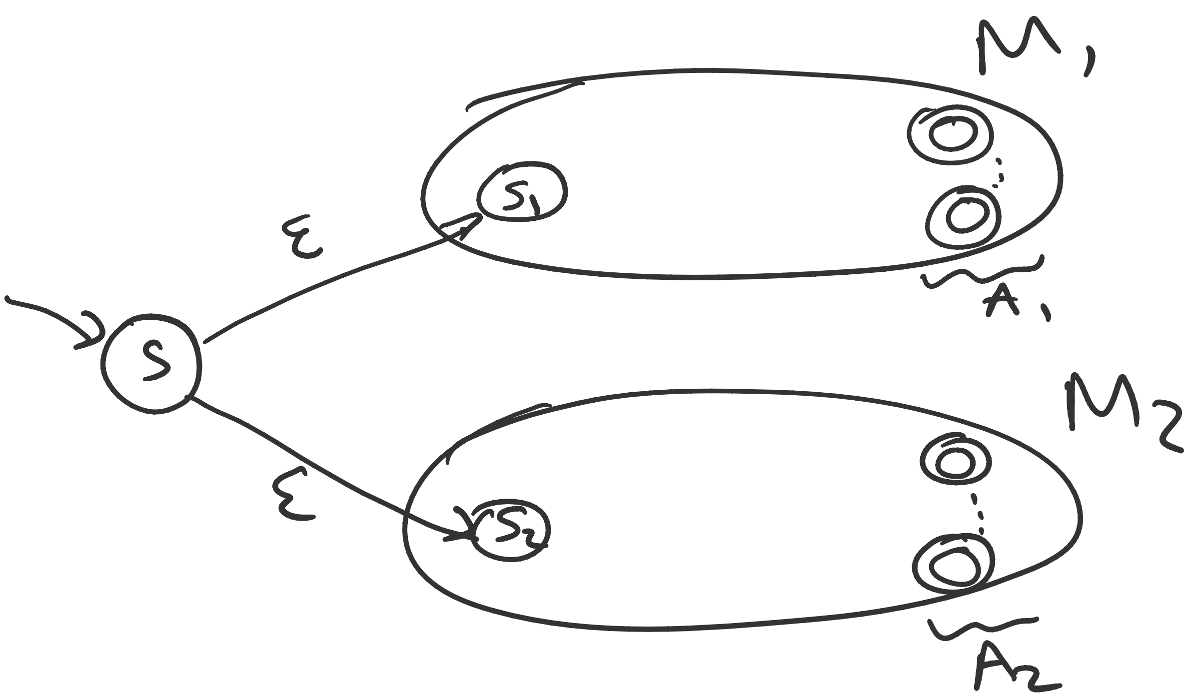


Induction step:

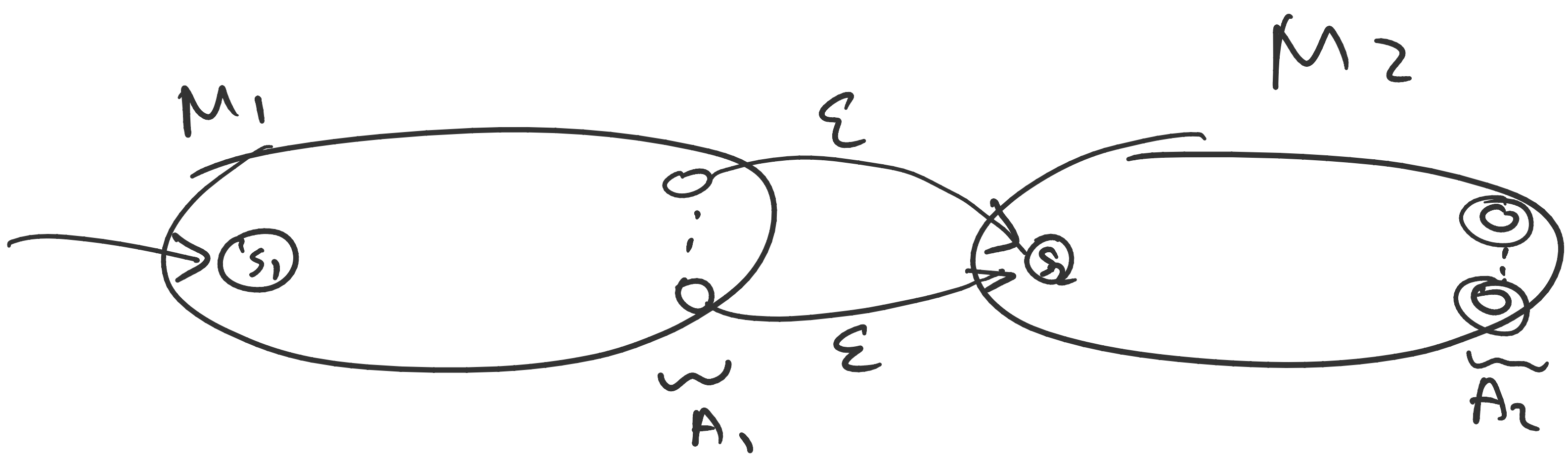
Suppose $L(r_1)$ is accepted by NFA $M_1 = (\Sigma, Q_1, s_1, A_1, \delta_1)$

$L(r_2)$ " " " $M_2 = (\Sigma, Q_2, s_2, A_2, \delta_2)$

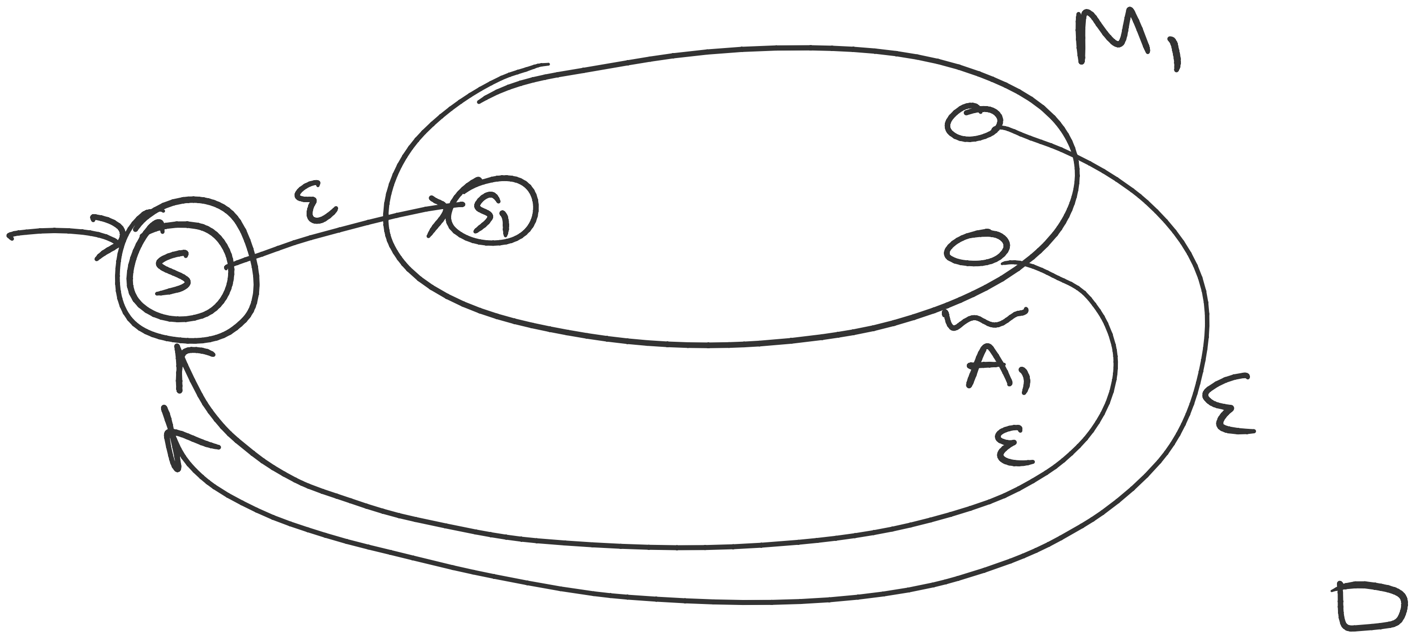
Then ① $r_1 + r_2$



② $r_1 r_2$

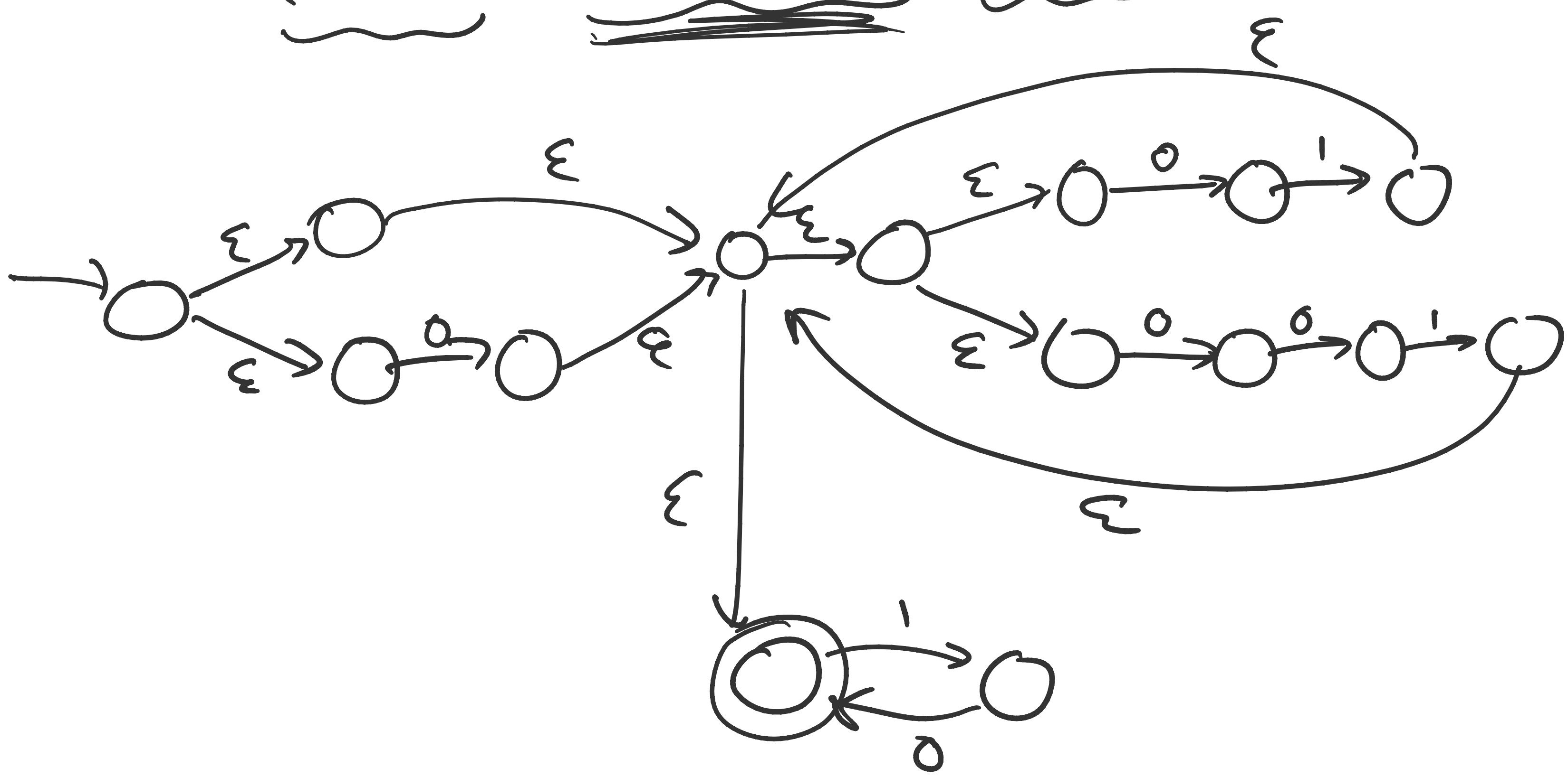


③ r_1^*



Ex

$$\underbrace{(\varepsilon + 0)}\ \underbrace{(01 + 001)^*}\ \underbrace{(10)^*}$$



NFA $\rightarrow$ DFA

Lemma If L is accepted by NFA M, then L is accepted by some DFA M'.

Pf: method - subset construction (i.e. power set)

idea - at any time,

idea - at any time, remember subset of all states you can be in

Given NFA $M = (\Sigma, Q, s, A, \delta)$

where $\delta: Q \times (\Sigma \cup \{\varepsilon\}) \to \mathcal{P}(Q)$,

Construct DFA $M' = (\Sigma, Q', s', A', \delta')$

where $\delta': Q' \times \Sigma \to Q'$:

$$Q' = \mathcal{P}(Q) \qquad (|Q'| = 2^{|Q|})$$

$$s' = \varepsilon\text{-reach}(s)$$

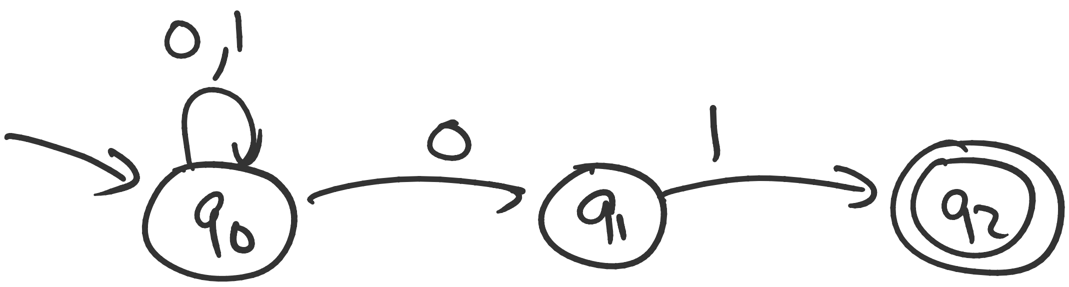
$$A' = \{ S \in Q' \mid S \cap A \neq \emptyset \}$$

$\forall S \in Q'$, $a \in \Sigma$, $$\delta'(S, a) = \bigcup_{q \in S} \delta^*(q, a)$$

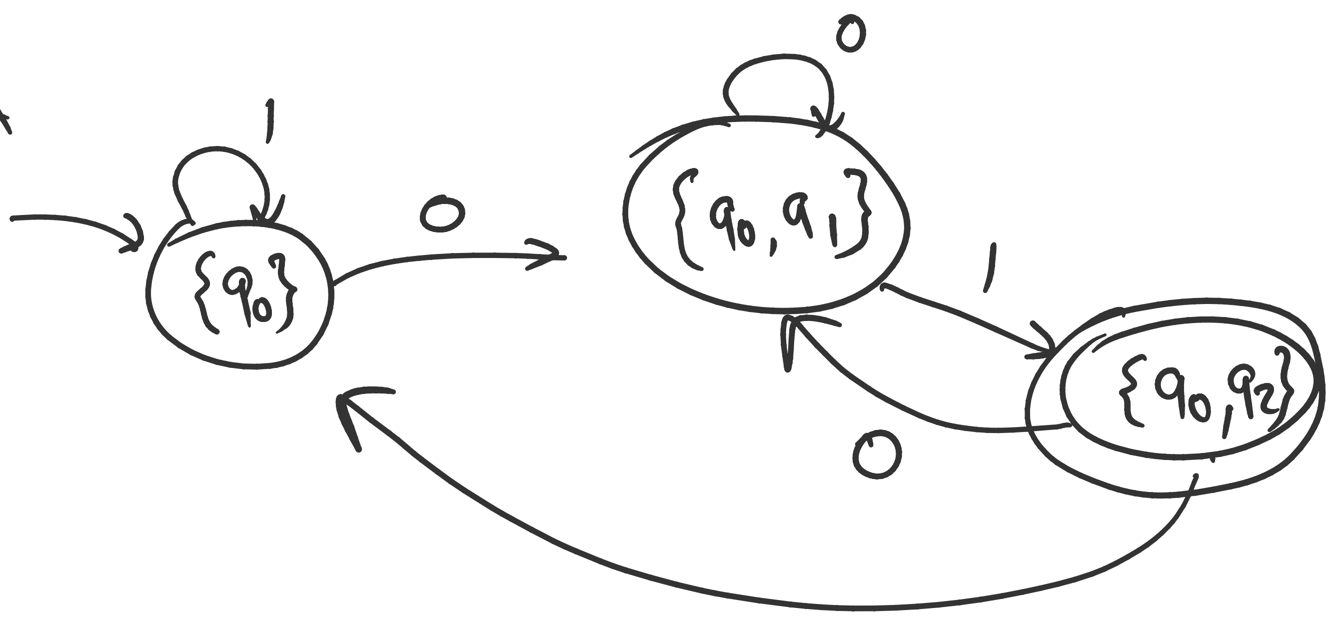
$\square$

Ex

NFA



DFA



DFA $\to$ Regular

Lemma If L is accepted by DFA M, then L is regular.

Pf Sketch: method - <u>state elimination</u>

Given DFA $M = (\Sigma, Q, s, A, \delta)$,

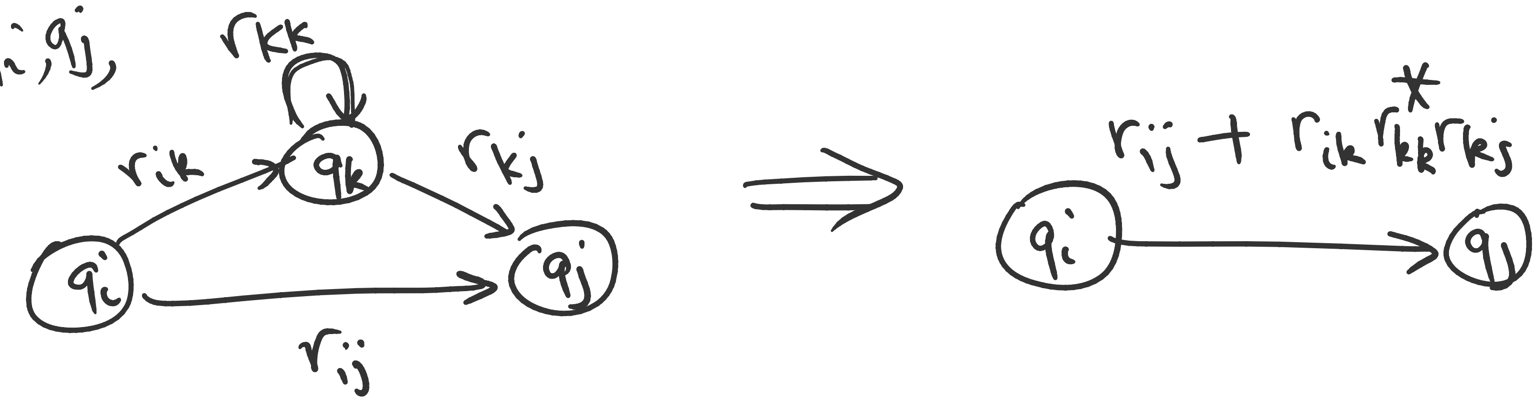
add new start state s', new final state f'

for each $q_k \in Q \setminus \{s', f'\}$,

remove q_k &

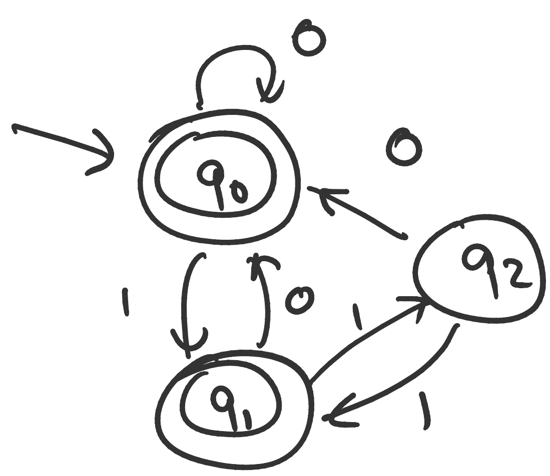
relabel transitions by this rule:

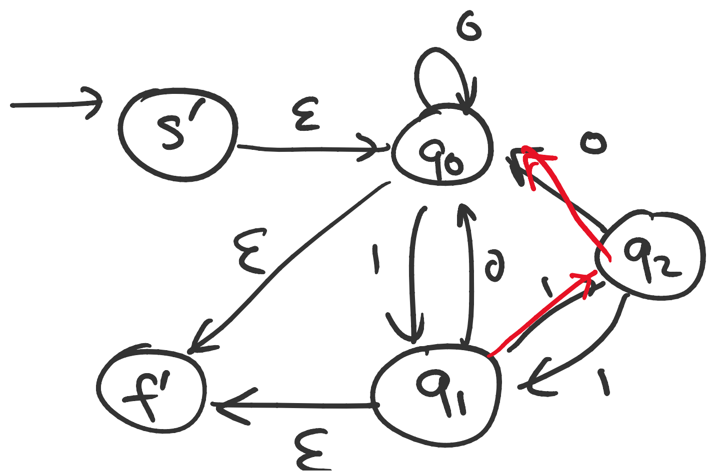
$\forall\ q_i, q_j,$



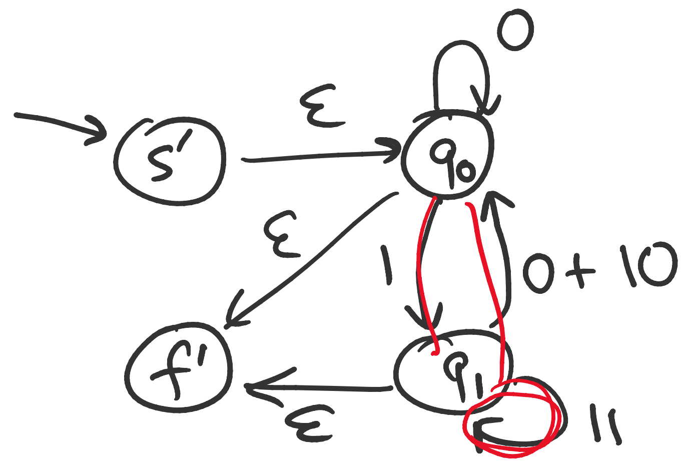
return label from s' to f'. $\square$

Ex

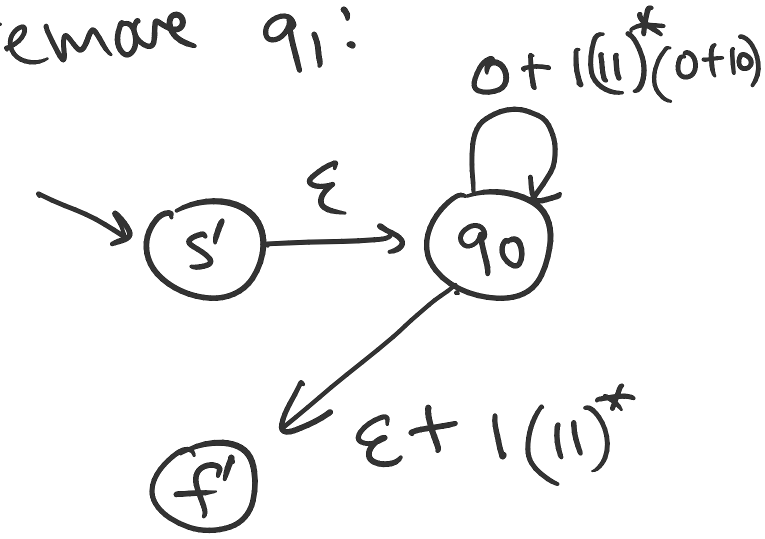




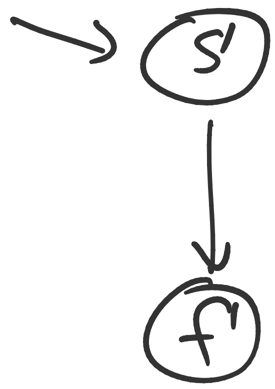
remove q_2:



remove q_1:



remove q_0:



$$\boxed{(0+1(11)^*(0+10))^*(\varepsilon+1(11)^*)}$$