Prove that each of the following problems is NP-hard.

1 Given an undirected graph G, does G contain a simple path that visits all but 374 vertices?

2 Given an undirected graph G, does G have a spanning tree with at most 374 leaves?

3 Recall that a 5-coloring of a graph G is a function that assigns each vertex of G a "color" from the set $\{0, 1, 2, 3, 4\}$, such that for any edge uv, vertices u and v are assigned different "colors". A 5-coloring is ***careful*** if the colors assigned to adjacent vertices are not only distinct, but differ by more than 1 (mod 5). Prove that deciding whether a given graph has a careful 5-coloring is NP-hard. (**Hint:** Reduce from the standard 5Color problem.)

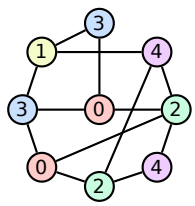

Figure 1: A careful 5-coloring.