January 24, 2019 10:58 AM

Last Time:

DFA

- $O(1)$ memory
- read input from L to R

Thm If L_1 accepted by DFA M_1,
L_2 accepted by DFA M_2,
$L_1 \cap L_2$ " " by some DFA.
$\overline{L_1}$ " " " " "

Cor can also do $L_1 \cup L_2$
& $L_1 \setminus L_2$

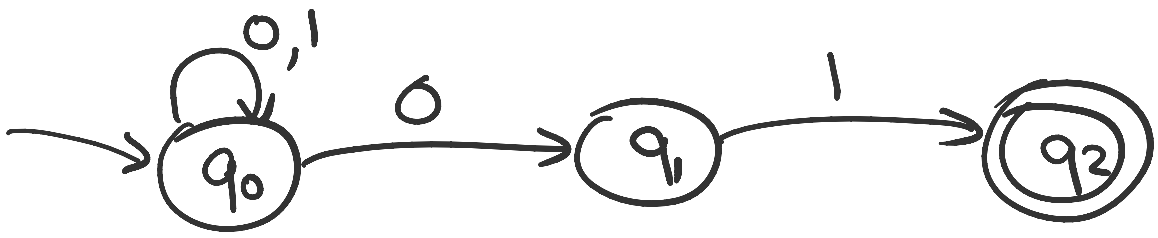
(because $L_1 \cup L_2 = \overline{\overline{L_1} \cap \overline{L_2}}$).

Will prove DFA $\leftrightarrow$ regular langs.

Nondeterministic Finite Automata (NFA)

- allow choices
- allow ε-transitions

Ex 1 all strings ending with 01



not valid DFA
but ok for NFA

e.g. $\delta(q_0, 0) = q_0$ or q_1?
$\delta(q_1, 0)$ is undef.

accept iff $\exists$ path leading to accept state

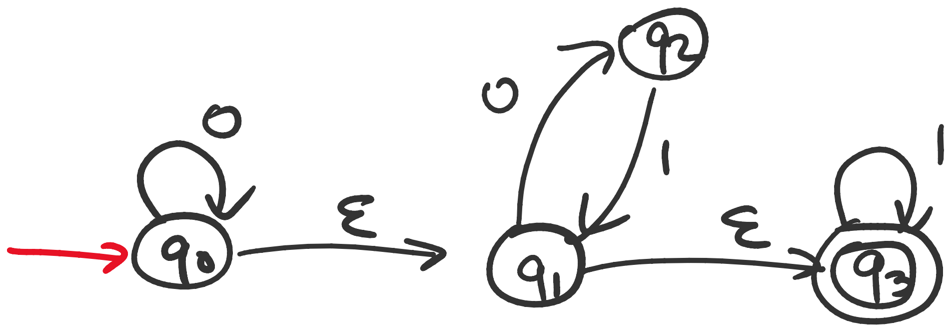
e.g. 001 $\quad q_0 \xrightarrow{0} q_0 \xrightarrow{0} q_1 \xrightarrow{1} q_2$

reject iff $\forall$ path, lead to non-accept state

not realistic machine!
(not obviously convertible to efficient program)

Ex2

$0^*(01)^*1^*$



(ε-transitions don't consume input)

Formal Def An NFA is $M = (Q, \Sigma, \delta, s, A)$

states
alphabet
transition fn
start state ($s \in Q$)
accept states ($A \subseteq Q$)

like before,

except

(before $\delta: Q \times \Sigma \to Q$)

$$\delta: Q \times (\Sigma \cup \{\varepsilon\}) \to \mathcal{P}(Q)$$

power set of Q
= { all subsets of Q }

$= \{$ all subsets of $Q\}$

Ex1 $\delta(q_0, 0) = \{q_0, q_1\}$

$\delta(q_1, 1) = \phi$

Ex2 $\delta(q_0, \varepsilon) = \{q_1\}$

Def Given $q \in Q$, define $\underline{\varepsilon\text{-reach}(q)}$ inductively:

(i) q is in ε-reach(q)

(ii) if q' is in ε-reach(q),
& $q'' \in \delta(q', \varepsilon)$,
then q'' is in ε-reach(q).

(Nothing else is in).

Ex2

$\varepsilon\text{-reach}(q_0) = \{q_0, q_1, q_3\}$

$\varepsilon\text{-reach}(q_1) = \{q_1, q_3\}$

Def Define $\underline{\text{extended transition fn}}$
$\delta^*: Q \times \Sigma^* \to \mathcal{P}(Q)$ inductively:

(i) $\delta^*(q, \varepsilon) = \varepsilon\text{-reach}(q)$

(ii) $$\delta^*(q, x) = \bigcup_{q' \in \varepsilon\text{-reach}(q)} \; \bigcup_{q'' \in \delta(q', a)} \delta^*(q'', y)$$

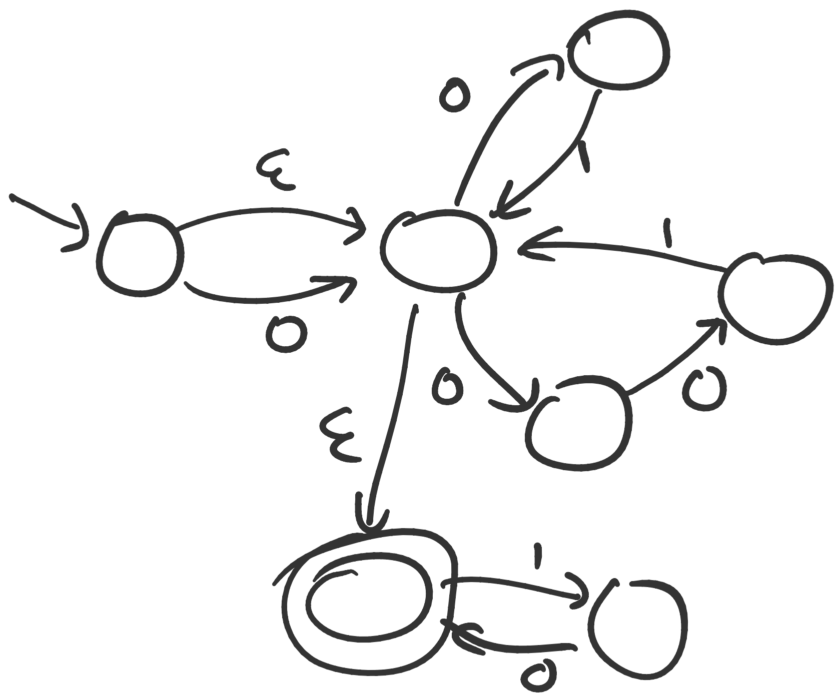
for $x = ay$
$(a \in \Sigma, y \in \Sigma^*)$

Define $L(M) = \{ x \in \Sigma^* \mid$

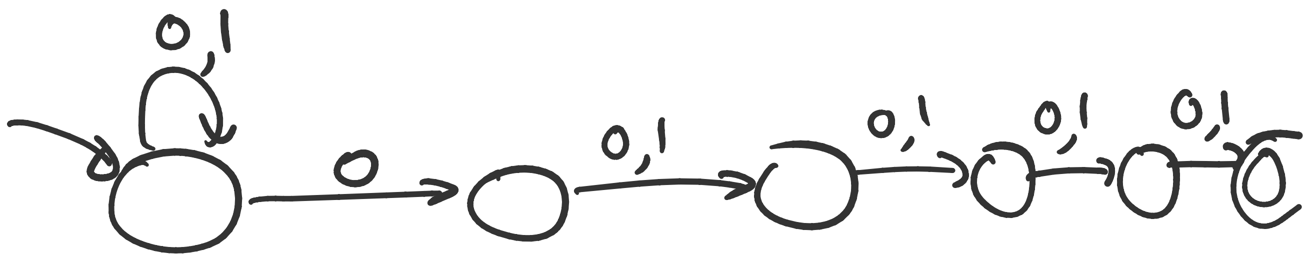
Define $L(M) := \{ x \in \Sigma^* \mid \delta^*(s, x) \cap A \neq \emptyset \}$

lang accepted by M

Ex a) $(\varepsilon + 0)(01 + 001)^*(10)^*$



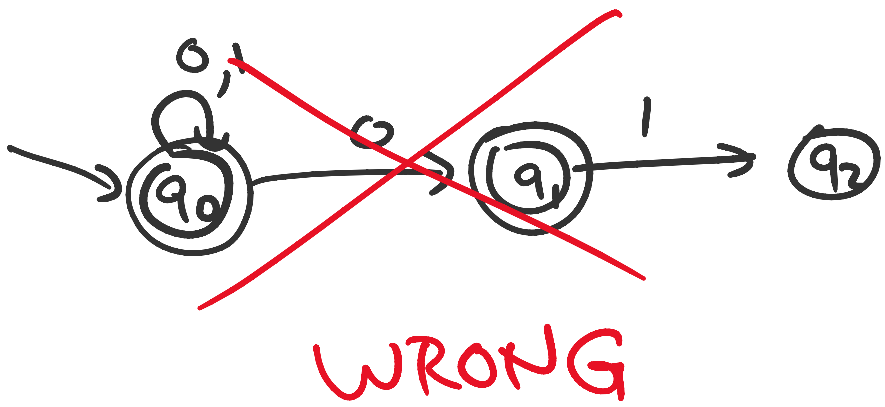
b) all strings whose 5^{th} symbol from right is a 0.



(any DFA needs 32 states!)

$- - 0 - - x\,x$

c) all strings <u>not</u> ending with 01

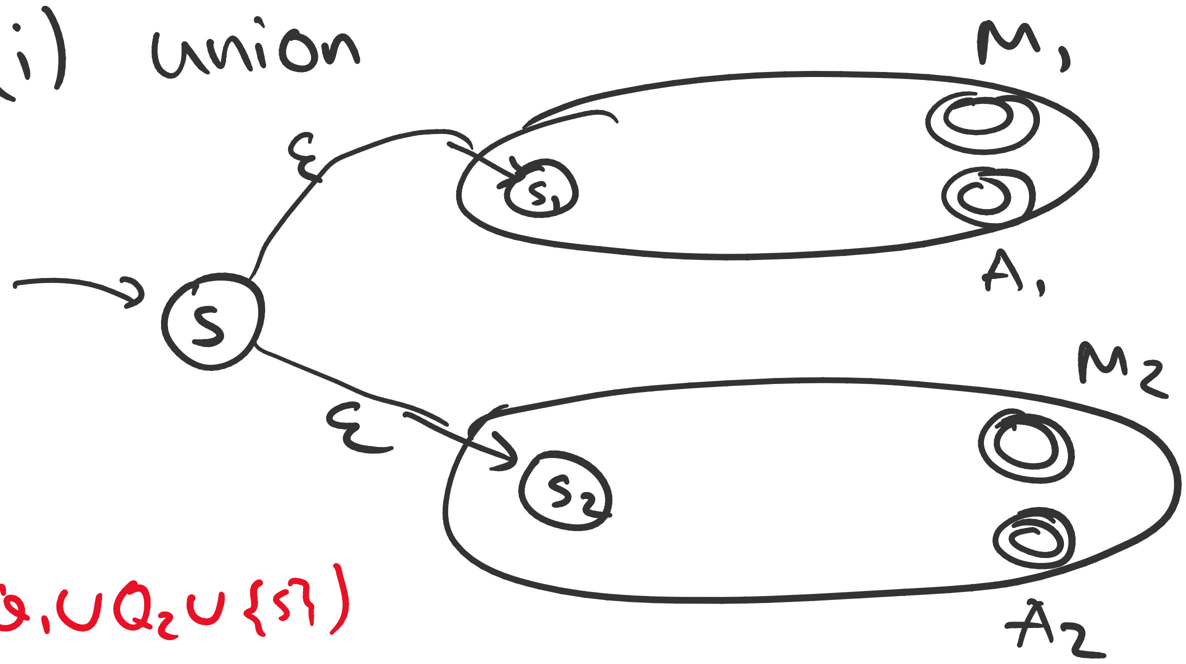


Regular → NFA

Thm If L_1 is accepted by NFA M_1,
L_2 " " " " M_2,
then (i) $L_1 \cup L_2$ is " " some NFA.
(ii) $L_1 L_2$ " " " "
(iii) L_1^* " " "

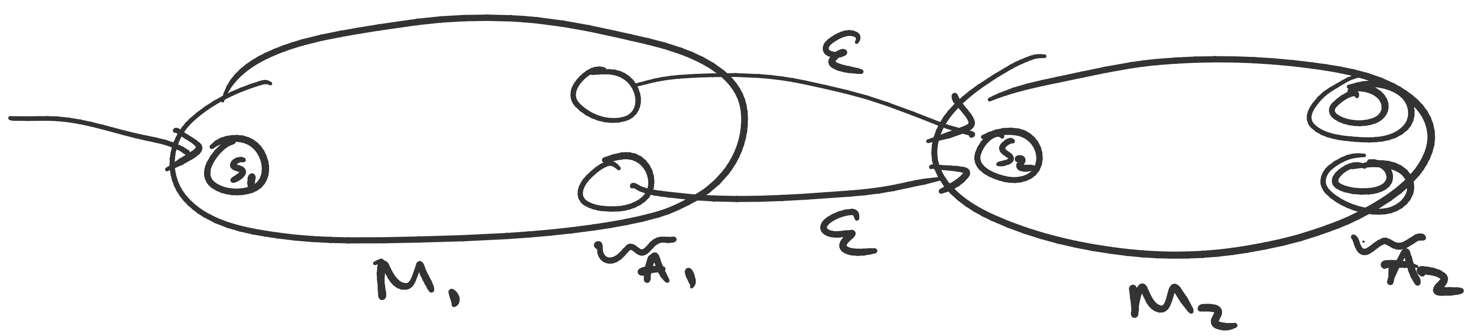
Pf: Given $M_1 = (Q_1, \Sigma, \delta_1, s_1, A_1)$
$M_2 = (Q_2, \Sigma, \delta_2, s_2, A_2)$

(i) union

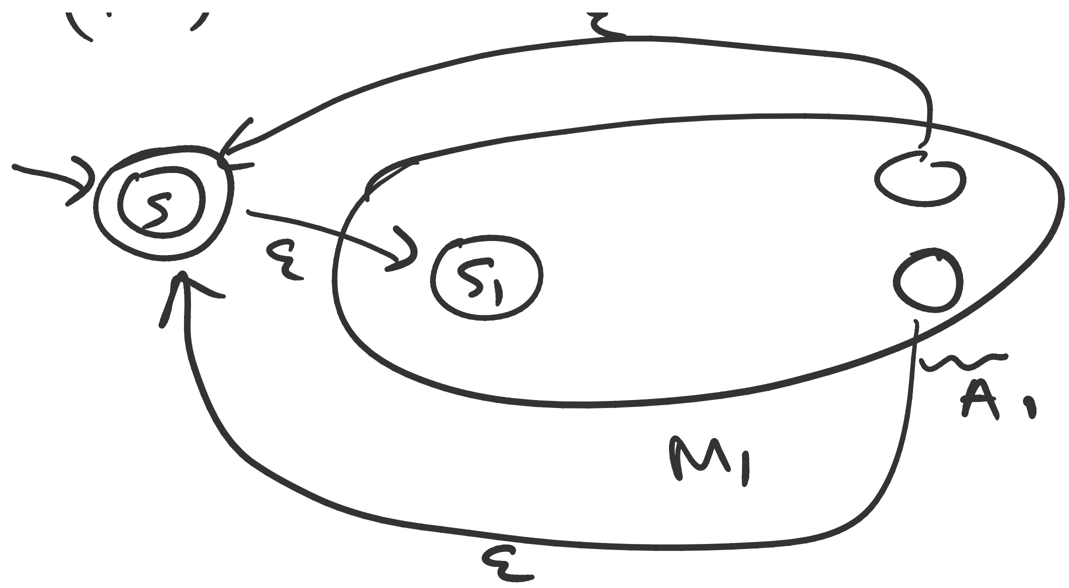


$(Q = Q_1 \cup Q_2 \cup \{s\})$

(ii) concat



(iii) star


s
ε
s₁
A₁
M₁
ε
ε