Every RSO has officers — pres, treasurer, ...

President's Club

↓

Non-President's Club <u>can't</u> be an RSO!

Barber [Russell]

Haircut Day

Licensed barber — cuts anyone's hair who doesn't cut their own

DON'T EXIST

<u>Cantor's Theorem:</u> Any set X

Any function $f: X \to 2^X$ subsets of X

f is <u>not</u> surjective

$\exists S \in 2^X$ s.t. $f(x) \neq S$ for all $x \in X$

$S \subseteq X$

<u>Proof:</u> Call $x \in X$ happy if $x \in f(x)$

sad $x \notin f(x)$

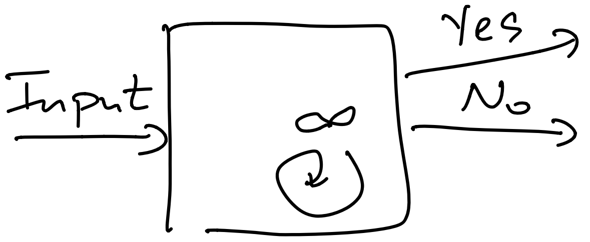
Let $S =$ all sad elements of $X = \{x \mid x \notin f(x)\}$

Spose for argument f <u>is</u> surjection

Then $\exists y \in X$ s.t. $f(y) = S$

<u>For all x:</u> $x \in S \iff x \in f(y) \iff x \notin f(x)$

So $y \in f(y) \iff y \notin f(y)$



source code machine

$$\text{SelfReject} = \{\langle M\rangle \mid M \text{ rejects } \langle M\rangle\}$$

Suppose there is a program SR s.t.

Given another source $\langle M\rangle$

If M reject $\langle M\rangle$ SR returns YES

If M accepts $\langle M\rangle$ or hangs on $\langle M\rangle$ SR returns NO

SR accepts $\langle M\rangle \iff M$ rejects $\langle M\rangle$

Set $M = SR$

SR accepts $\langle SR\rangle \iff$ SR rejects $\langle SR\rangle$

Contradiction!

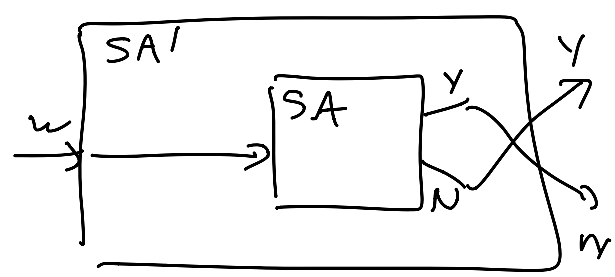
$$\text{SelfAccept} = \{\langle M\rangle \mid M \text{ accepts } \langle M\rangle\}$$

~~Suppose~~ there is a machine SA that decides SelfAccept

Build new machine SA':

SA'(w):

```
ok ← SA(w)
return ¬ok
```



SA' accepts $\langle M\rangle \iff M$ does not accept $\langle M\rangle$

SA' accepts $\langle SA'\rangle \iff$ SA' does not accept $\langle SA'\rangle$

Halting Problem: Given $\langle M \rangle$ and w

Does M halt if w is the input to M?

Collatz(n):

```
if n=1 halt
else if n even
    Collatz(n/2)
else
    Collatz(3n+1)
```

Spose H decides $HALT = \{\langle M \rangle \circ w \mid M \text{ halts on } w\}$

SH(x):

```
first check x = <M> for some M
    else reject
if H(x, x)
    return True
else
    return FALSE
```

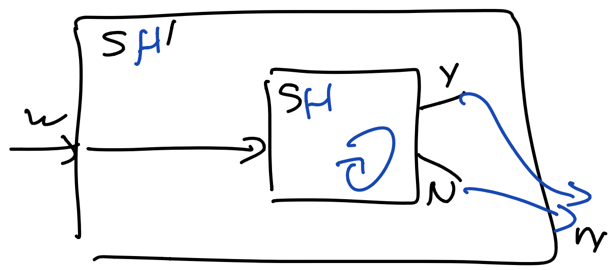
Self Halt $= \{\langle M \rangle \mid M \text{ halts on } \langle M \rangle\}$

~~Suppose~~ there is a machine SH that decides Self Halt

Build new machine SA':

SH'(w):

```
run SH(w)
return FALSE
```



SH' accepts $\langle M \rangle \iff M$ does not accept $\langle M \rangle$

SH' accepts $\langle SA' \rangle \iff SA'$ does not accept $\langle SA' \rangle$