CS 374 Lab 12: Backtracking

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A **subsequence** of a sequence (for example, an array, linked list, or string), obtained by removing zero or more elements and keeping the rest in the same sequence order. A subsequence is called a **substring** if its elements are contiguous in the original sequence. For example:

- SUBSEQUENCE, UBSEQU, and the empty string ϵ are all substrings of the string SUBSEQUENCE;
- SBSQNC, UEQUE, and EEE are all subsequences of SUBSEQUENCE but not substrings;
- QUEUE, SSS, and FOOBAR are not subsequences of SUBSEQUENCE.

Describe recursive backtracking algorithms for the following problems. *Don't worry about running times.*

1. Given an array $A[1..n]$ of integers, compute the length of a **longest increasing subsequence**. A sequence $B[1..\ell]$ is *increasing* if $B[i] > B[i-1]$ for every index $i \geq 2$. For example, given the array

   $$\langle 3, \underline{\mathbf{1}}, \underline{\mathbf{4}}, 1, \underline{\mathbf{5}}, 9, 2, \underline{\mathbf{6}}, 5, 3, 5, \underline{\mathbf{8}}, \underline{\mathbf{9}}, 7, 9, 3, 2, 3, 8, 4, 6, 2, 7 \rangle$$

   your algorithm should return the integer 6, because $\langle 1, 4, 5, 6, 8, 9 \rangle$ is a longest increasing subsequence (one of many).

2. Given an array $A[1..n]$ of integers, compute the length of a **longest decreasing subsequence**. A sequence $B[1..\ell]$ is *decreasing* if $B[i] < B[i-1]$ for every index $i \geq 2$. For example, given the array

   $$\langle 3, 1, 4, 1, 5, \underline{\mathbf{9}}, 2, \underline{\mathbf{6}}, 5, 3, \underline{\mathbf{5}}, 8, 9, 7, 9, 3, 2, 3, 8, \underline{\mathbf{4}}, 6, \underline{\mathbf{2}}, 7 \rangle$$

   your algorithm should return the integer 5, because $\langle 9, 6, 5, 4, 2 \rangle$ is a longest decreasing subsequence (one of many).

3. Given an array $A[1..n]$ of integers, compute the length of a **longest alternating subsequence**. A sequence $B[1..\ell]$ is *alternating* if $B[i] < B[i-1]$ for every even index $i \geq 2$, and $B[i] > B[i-1]$ for every odd index $i \geq 3$. For example, given the array

   $$\langle \underline{\mathbf{3}}, \underline{\mathbf{1}}, \underline{\mathbf{4}}, \underline{\mathbf{1}}, \underline{\mathbf{5}}, 9, \underline{\mathbf{2}}, \underline{\mathbf{6}}, \underline{\mathbf{5}}, 3, 5, \underline{\mathbf{8}}, 9, \underline{\mathbf{7}}, \underline{\mathbf{9}}, \underline{\mathbf{3}}, 2, 3, \underline{\mathbf{8}}, \underline{\mathbf{4}}, \underline{\mathbf{6}}, \underline{\mathbf{2}}, \underline{\mathbf{7}} \rangle$$

   your algorithm should return the integer 17, because $\langle 3, 1, 4, 1, 5, 2, 6, 5, 8, 7, 9, 3, 8, 4, 6, 2, 7 \rangle$ is a longest alternating subsequence (one of many).