Sorted except k corrupted

$O(k \log \frac{n}{k})$

~~if A[i] > A[i-1]~~

~~A[i] is corrupted~~

if $x < A[m]$

return Search(lo, m-1, x, k) $\vee$ Search(m+1, hi, x, k-1)

$$T(n,k) = \begin{cases} 0 & \text{if } n=0 \text{ or } k=1 \\ T(\frac{n}{2}, k) + T(\frac{n}{2}, k-1) \end{cases}$$

$$T(n,k) = O((\log n)^{k+1})$$

if window is sorted

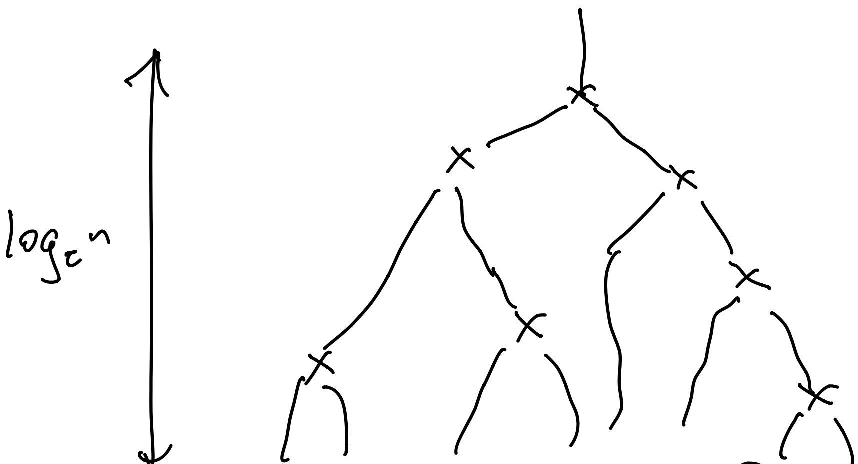
if $x < A[m]$

Search(lo, m-k-1, x)

⋮

if not

Search(lo, m-k-1, x) $\vee$ Search(m+k+1, hi, x)



k bad values

$\leq k$ nodes in recursion tree with 2 kids

Overall $O(k^2 \log n)$ time

Algorithms / TMs / programs → text $\langle M \rangle$, device M → return TRUE (Accept), return FALSE (REJECT), ∞ loop (HANG)

Decisionproblems — answer is TRUE or FALSE

For any machine M defines 3 languages

Accept(M)

Reject(M)

Hang(M)

M **decides** $L \subseteq \Sigma^*$ iff

$x \in L \iff M$ accepts x ← M accepts L

$x \notin L \iff M$ rejects x ← M rejects $\Sigma^* \setminus L$

SELFHALT $:= \{\langle M \rangle \mid M$ halts on $\langle M \rangle\}$

Suppose SH **decides** SELFHALT

WTF(x):

```
if SH(x) = TRUE
    loop forever
else
    halt
```

WTF halts on $\langle M \rangle \iff$ SH rejects $\langle M \rangle \iff M$ hangs on $\langle M \rangle$

$\boxed{\text{Set } M = \text{WTF}}$

WTF halts on $\langle$WTF$\rangle \iff$ WTF hangs on $\langle$WTF$\rangle$!!

So SH doesn't exist!

1	0. 0 0 0 0 0 0 . .
2	0. 1 0 1 1 0 1 . . .
3	0. 1 1 0 1 0 0 1
4	0. 0 0 1 0 1
⋮	⋮

0.1111. . . .

Cantor's Thm: No surjection $X \to 2^X$

$\text{HALT} = \{\langle M\rangle \# w \mid M \text{ halts on } w\}$

$\#$ ← separator

$(\langle M\rangle, w)$

The following algo <u>accepts</u> HALT:

```
AcceptHalt(<M>, w):
    run M on w
    return True
```

Suppose ~~H~~ <u>decides</u> HALT:

```
WTF(x):
    if H(x,x)
        loop forever
    else
        halt
```

WTF halts on <WTF> $\iff$ WTF hangs on <WTF>

```
SelfHalt(<M>):
    return HALT(<M>,<M>)
```

$\text{AlwaysHalt} = \{\langle M\rangle \mid M \text{ halts on } \underline{\text{all}} \text{ inputs}\}$

$\text{HALT}(M) = \Sigma^*$

Reduce from HALT

Suppose ~~AH~~ decides ALWAYSHALT

$H(\langle M\rangle, w)$:

Write encoding $\langle M_w\rangle$ of following:

$M_w(x)$:

Run M on w

return $AH(\langle M_w\rangle)$

If M halts on w

$\longrightarrow$ M_w halts on every input x.

$\longrightarrow$ $\langle M_w\rangle \in$ ALWAYSHALTS

$\longrightarrow$ AH accepts $\langle M_w\rangle$

$\longrightarrow$ H accepts $\langle M\rangle, w$

If M hangs on w

$\longrightarrow$ M_w hangs on <u>all</u> inputs x

$\longrightarrow$ $\langle M_w\rangle \notin$ ALWAYSHALT

$\longrightarrow$ AH rejects $\langle M_w\rangle$

$\longrightarrow$ H rejects $\langle M\rangle, w$

$\mathcal{L}$ = family of languages — "finite" "regular" "decidable" "empty"

$\text{AcceptIn}(\mathcal{L}) = \{\langle M\rangle \mid \text{Accept}(M) \in \mathcal{L}\}$

Rice's Thm:

If $\exists$ machine Y s.t. $\text{Accept}(Y) \in \mathcal{L}$

$\exists$ machine N s.t. $\text{Accept}(N) \notin \mathcal{L}$

Then $\text{AcceptIn}(\mathcal{L})$ is undecidable

$\text{FINITE} = \{\langle M\rangle \mid \text{Accept}(M) \text{ is finite}\}$

"return False" accepts $\emptyset \in \mathcal{L}$.

"return true" accepts $\Sigma^* \notin \mathcal{L}$

$\text{Empty} = \{\langle M \rangle \mid \text{Accept}(M) = \emptyset\}$

$\text{AcceptSelfSelf} = \{\langle M \rangle \mid M \text{ accepts } \langle M \rangle \langle M \rangle\}$