Midterm 1 — week from Monday

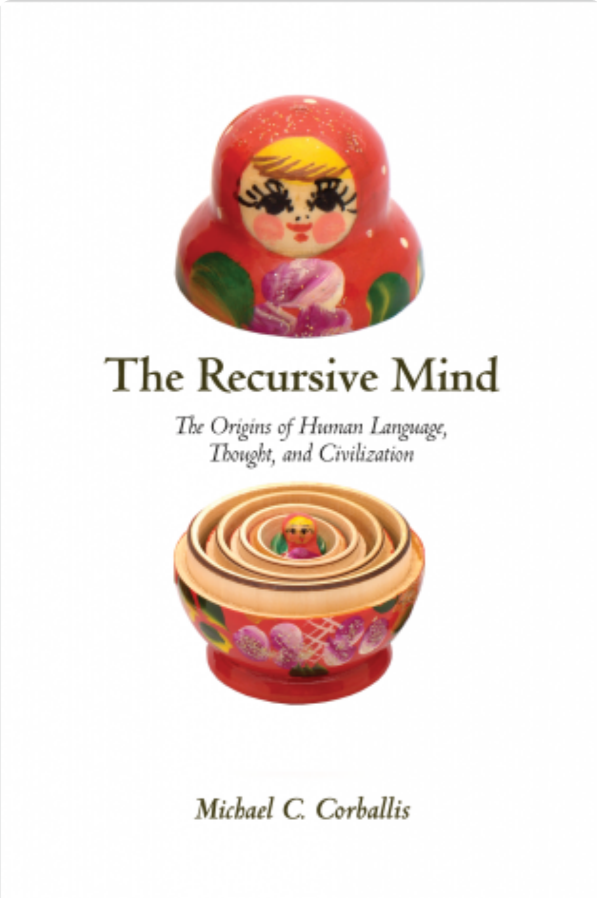
Conflict: week from Tuesday [signup form]

DRES — reserve soon!

Start on HW3 already

Regular: built from single strings
- Sequencing A·B
- Branching A+B
- Repetition A*

Context-free:
- All of the above
+ Recursion




## Evidence of recursion in tool use

doi:10.1017/S0140525X11001865

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**Abstract:** We discuss the discovery of technologies involving knotted netting, such as textiles, basketry, and cordage, in the Upper Paleolithic. This evidence, in our view, suggests a new way of connecting toolmaking and syntactic structure in human evolution, because these technologies already exhibit an "infinite use of finite means," which we take to constitute the key transition to human cognition.

nature Vol 440|27 April 2006|**doi:10.1038/nature04675**

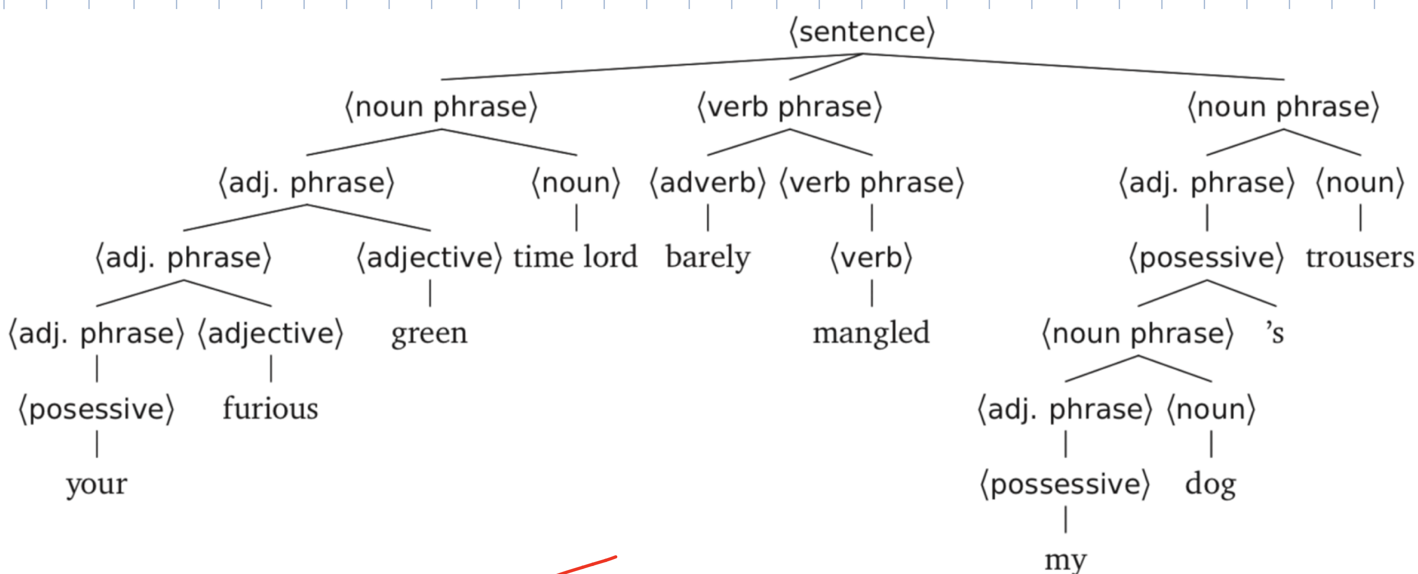
LETTERS

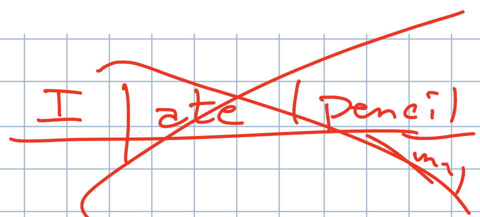
# Recursive syntactic pattern learning by songbirds

Timothy Q. Gentner[1]†, Kimberly M. Fenn[2], Daniel Margoliash[1,2] & Howard C. Nusbaum[2]

⟨sentence⟩ → ⟨noun phrase⟩⟨verb phrase⟩⟨noun phrase⟩
⟨noun phrase⟩ → ⟨adjective phrase⟩⟨noun⟩
⟨adj. phrase⟩ → ⟨article⟩ | ⟨possessive⟩ | ⟨adjective phrase⟩⟨adjective⟩
⟨verb phrase⟩ → ⟨verb⟩ | ⟨adverb⟩⟨verb phrase⟩

⟨noun⟩ → dog | trousers | daughter | nose | homework | time lord | pony | ⋯
⟨article⟩ → the | a | some | every | that | ⋯
⟨possessive⟩ → ⟨noun phrase⟩’s | my | your | his | her | ⋯
⟨adjective⟩ → friendly | furious | moist | green | severed | timey-wimey | little | ⋯
⟨verb⟩ → ate | found | wrote | killed | mangled | saved | invented | broke | ⋯
⟨adverb⟩ → squarely | incompetently | barely | sort of | awkwardly | totally | ⋯



$S \to A$
$S \to B$
$A \to 0A$
$A \to 0C$ $\Big\}$ $\to A \to 0^+C$
$B \to B1$
$B \to C1$
$C \to \varepsilon$
$C \to 0C1$

$\Sigma$ – alphabet $= \{0,1\}$
terminals

$\Gamma$ – non-terminals $= \{S, A, B, C\}$

production rules $A \to w$

$A \in \Gamma \quad w \in (\Sigma \cup \Gamma)^*$

stating non-terminal $S$

$L(A)$ = set of strings generated by $A$

$G = (\Sigma, \Gamma, R, S) \quad L(G) = L(S)$

| | | |
|---|---|---|
| $S$ | $\to$ | $A \mid B$ |
| $A$ | $\to$ | $0A \mid 0C$ |
| $B$ | $\to$ | $B1 \mid C1$ |
| $C$ | $\to$ | $\varepsilon \mid 0C1$ |

Backus-Naur form
BNF

$S \rightsquigarrow$

$00C1BAC0 \longrightarrow 00\underbrace{0C1}1BAC0 \rightsquigarrow 0001101100000$

$xAy \rightsquigarrow xwy$
produces immediately

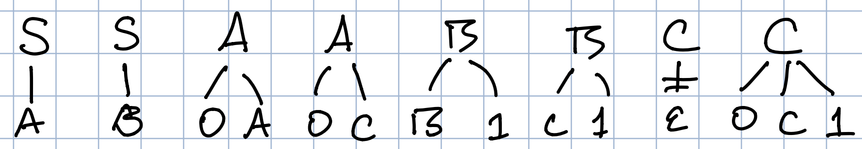
rule
$A \to w$

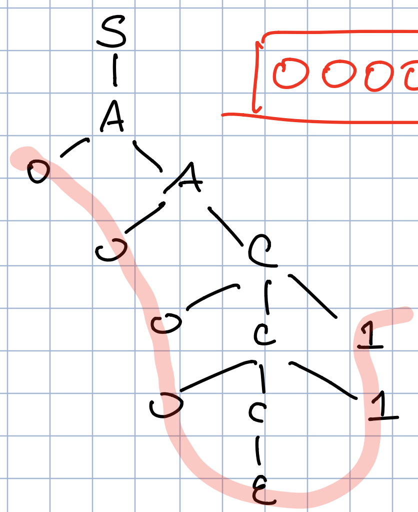
$S \rightsquigarrow^* w$ produces

| | | |
|---|---|---|
| $S$ | $\to$ | $A \mid B$ |
| $A$ | $\to$ | $0A \mid 0C$ |
| $B$ | $\to$ | $B1 \mid C1$ |
| $C$ | $\to$ | $\varepsilon \mid 0C1$ |

$\to \{0^m 1^n \mid n \neq m\}$ both $\geq 0$
$\to \{0^m 1^n \mid n < m\} \geq 0$
$\to \{0^m 1^n \mid n > m\} \geq 0$
$\to \{0^m 1^n \mid m = n \geq 0\}$

$S \to A \mid B$
$A \to 0A \mid 0C$
$B \to B1 \mid C1$
$C \to \varepsilon \mid 0C1$





$\boxed{000011}$

$0^*1^*$

$S \to \varepsilon \mid 0S \mid S1$

$S \to AB$
$A \to \varepsilon \mid 0A$
$B \to \varepsilon \mid 1B$

$\{0^n1^n0^n \mid n \geq 0\}$ is <u>not</u> CFL

$\boxed{C \to \varepsilon \mid 0C1} = \{0^n1^n \mid n \geq 0\}$

<u>Lemma</u>: $C \rightsquigarrow^* 0^n1^n$ for all $n \geq 0$

<u>Proof</u>: Let $n$ be arbitrary non-neg int

Assume $C \rightsquigarrow^* 0^m1^m$ for all $m < n$

Two cases:

- $n = 0$ : $0^n1^n = \varepsilon$ $\quad C \to \varepsilon$ ✓
- $n > 1$ $\quad C \to 0C1 \xrightarrow{IH} 0(0^{n-1}1^{n-1})1 = 0^n1^n$ ✓

Thus $C \rightsquigarrow^* 0^n1^n$

<u>Lemma</u>: For all $w \in L(C)$, $w = 0^n1^n$ for some $n \geq 0$

Fix $w \in L(C)$

Assume for all $x \in L(C)$ with $|x| < |w|$, $x = 0^m1^m$ for some $m \geq 0$

Two cases (First production)

- $C \to \varepsilon \Rightarrow w = \varepsilon = 0^0 1^0$ ✓
- $C \rightsquigarrow 0C1 \Rightarrow w = 0x1$ for some $x \in L(C)$

  $\overset{IH}{\Rightarrow} w = 0(0^m1^m)1$ for some $m \geq 0$

  $= 0^{m+1}1^{m+1}$

Thus $w = 0^n1^n$ for some $n \geq 0$

## Chomsky Normal Form

$S \to \varepsilon$ maybe

$\left.\begin{array}{l} A \to a \\ A \to BC \end{array}\right\}$ otherwise

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Strings $w$ with $\#(0,w) = \#(1,w)$
$\in (0+1)^*$

$\cancel{S \to 01S \mid 0S1 \mid S01 \mid S10 \mid 1S0 \mid 10S}$

$\cancel{S \to A \mid B}$

$\cancel{A \to 0 \mid \varepsilon \mid AB}$

$\cancel{B \to 1 \mid \varepsilon \mid AB}$

$\cancel{S \to A \mid B}$

$\cancel{A \to 0A1 \mid 0B1 \mid 01}$ $\quad 0 \sim 1$

$\cancel{B \to 1B0 \mid 1A0 \mid 10}$ $\quad 1 \sim 0$

$S \to \varepsilon \mid 0S1 \mid 1S0 \mid SS$

$$\boxed{\begin{array}{l} S \to \varepsilon \mid AS \\ A \to 0S1 \mid 1S0 \end{array}}$$

— $\#0 = \#1$

~ start & end diff and $\#0 = \#1$

$S \to \varepsilon \mid (S) \mid SS$