Admin:

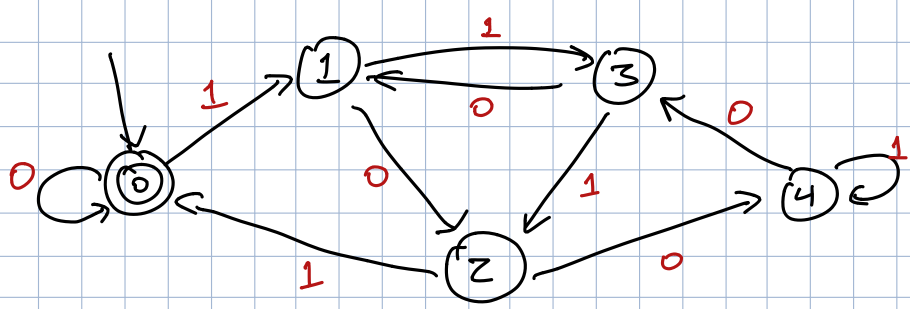
- Please communicate preferences for OH to TAs in lab
- Please submit only HW1.3 for HW1.3 (for example)

0010 → 2
11011 → 2
0011 → 3

00101 → 5
110111 → 5
00111 → 2

$\delta^*(s,w) = \delta^*(s,x) \implies$ for all $y \in \Sigma^*$

$wy \in L \iff xy \in L$



z is a distinguishing suffix for x, y

iff $xz \in L$ and $yz \notin L$

or

$xz \notin L$ and $yz \in L$

$\implies$ x and y lead to different states in EVERY DFA for L.

FOOLING SET F for L

$\forall x, y \in F$ where $x \neq y$

$\exists z \in \Sigma^*$

s.t. $xz \in L \iff yz \notin L$

Example: $L = \{\text{binary mult of } 5\}$

$F = \{0, 1, 10, 11, 100\}$

$$\begin{array}{ll} 101 & = 5 \\ 1101 & = 13 \end{array} \qquad \begin{array}{l} 100 \\ 1010 \end{array} \quad \begin{array}{l} \times \\ \checkmark \end{array} \qquad \begin{array}{l} 0\,\varepsilon \\ 11\,\varepsilon \end{array} \quad \begin{array}{l} \checkmark \\ \times \end{array}$$

$$\begin{array}{ll} 101z & = 5 \cdot 2^{|z|} + \langle z \rangle \\ 1010z & = 10 \cdot 2^{|z|} + \langle z \rangle \end{array}$$

If F is a fooling set for L

Any DFA for L has $\geq |F|$ states.

But what if F is infinite?!

$L = \{0^n 1^n \mid n \geq 0\} = \{\varepsilon, 01, 0011, 000111, 00001111, \ldots\}$

Let $F = 0^*$

Let x and y be arbitrary distinct strings in F

We can write $x = 0^i$ and $y = 0^j$ where $i \neq j$

Let $z = 1^i$

$xz = 0^i 1^i \in L$

$yz = 0^j 1^i \notin L$ because $i \neq j$

So x, y have dist. suffix

So F is a fooling set for L.

Because F is infinite, L cannot be regular

$L_2 = \{w\#w^R \mid w \in \Sigma^*\}$ = ~~even~~ odd length palindromes with # in middle

Let $F = 0^*$

Let x, y be arb. distinct strings in F

So $x = 0^i$ and $y = 0^j$ for some $i \neq j$

Let $z = \#0^i$

$xz \in L$ because $xz = 0^i \# 0^i$

$yz \notin L$ because $yz = 0^j \# 0^i$

So z dist suffix

So F fools L

So L not regular

$L_3 = \{ww^R \mid w \in \Sigma^*\}$

Let $F = 1^*00$

$F = (01)^*1$

01011 010

0101010111 010

$L_4 = \{www \mid w \in \Sigma^*\}$ 011011011

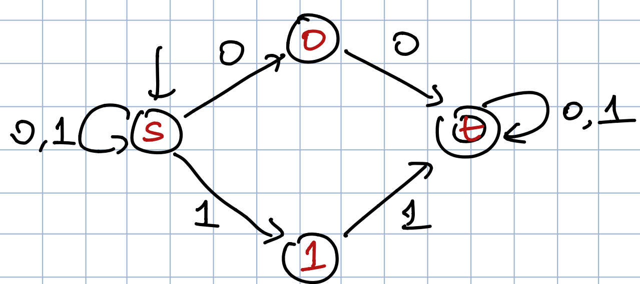
$F = \cancel{(00)^*}\ 0^*1$

00000001 00000001 0000001

0000 00000000001

$L_4 \cap 0^*10^*10^*1$

NFA



0011

$\delta(s,0) = \{s, 0\}$

$\delta^*(s,00) = \{s, 0, t\}$

$\delta^*(s,001) = \{s, 1, t\}$

$\delta^*(s,0011) = \{s, 1, t\}$

NFA <u>accepts</u> a string w if there is a seq of transitions

$$s \xrightarrow{a_1} q_1 \xrightarrow{a_2} q_2 \xrightarrow{a_3} \cdots \xrightarrow{a_n} q_\ell \in A$$

$$a_1 a_2 \cdots a_n = w$$

$NFA = (\Sigma, Q, s, A, \delta)$

Σ — input alphabet

Q — states

s — start state $\in Q$

$A \subseteq Q$ — accepting

$\delta : Q \times \Sigma \to 2^Q$ transition

$\delta^* : Q \times \Sigma^* \to 2^Q$

$$\delta^*(q,w) = \begin{cases} \{q\} & \text{if } w = \varepsilon \\ \bigcup_{p \in \delta(q,a)} \delta^*(p,x) & \text{if } w = ax \end{cases}$$