

Design Turing machines  $M = (Q, \Sigma, \Gamma, \delta, \text{start}, \text{accept}, \text{reject})$  for each of the following tasks, either by listing the states  $Q$ , the tape alphabet  $\Gamma$ , and the transition function  $\delta$  (in a table), or by drawing the corresponding labeled graph.

Each of these machines uses the input alphabet  $\Sigma = \{1, \#\}$ ; the tape alphabet  $\Gamma$  can be any superset of  $\{1, \#, \square, \triangleright\}$  where  $\square$  is the blank symbol and  $\triangleright$  is a special symbol marking the left end of the tape. Each machine should **reject** any input not in the form specified below.

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1. On input  $1^n$ , for any non-negative integer  $n$ , write  $1^n \# 1^n$  on the tape and **accept**.
2. On input  $\#^n 1^m$ , for any non-negative integers  $m$  and  $n$ , write  $1^m$  on the tape and **accept**. In other words, delete all the  $\#$ s and shift the  $1$ s to the start of the tape.
3. On input  $\# 1^n$ , for any non-negative integer  $n$ , write  $\# 1^{2^n}$  on the tape and **accept**. [*Hint: Modify the Turing machine from problem 1.*]
4. On input  $1^n$ , for any non-negative integer  $n$ , write  $1^{2^n}$  on the tape and **accept**. [*Hint: Use the three previous Turing machines as subroutines.*]