CS 374 Lab 22: Reductions, Undecidabile and Non-r.e. languages

Date: April 15, 2016.

Problem 1. [Category: Proof] For each of the languages below indicate whether it is (a) decidable (a.k.a. *recursive*), (b) recognizable (a.k.a. *recursively enumerable*) but not decidable, or (c) not recognizable.

You may use the following facts:

- the language SelfReject $= \{\langle M \rangle \mid M \text{ does not accept } \langle M \rangle\}$ is not recursively enumerable;
- the language Accept $= \{\langle M, w \rangle \mid M \text{ accepts } w\}$ is recursively enumerable but not decidable.

- $L_1 = \{\langle M, w \rangle \mid M \text{ does not halt on input } w\}$.
- $L_2 = \{\langle M, w, n \rangle \mid M \text{ accepts } w \text{ within } n \text{ steps}\}$.
- $L_3 = \{\langle M \rangle \mid M \text{ accepts } \langle M \rangle\}$.

Problem 2. [Category: Proof] We saw in class that the language Accept $= \{\langle M, w \rangle \mid M \text{ accepts } w\}$ is undecidable. Use a reduction to argue that the language $L = \{\langle M_1, M_2, w \rangle \mid$ exactly one of M_1 and M_2 accepts $w\}$ is undecidable.

Problem 3. [Category: Proof] Can you show that L from the previous problem is not recognizable? *Hint:* In the previous problem you showed Accept $\leq_m L$. Show that $\overline{\text{Accept}} \leq_m L$ using similar ideas, and then use the fact that $\overline{\text{Accept}}$ is not recognizable to complete the proof.