
CS 374 LAB 17: GRAPH DECOMPOSITION

Date: March 18, 2016.

The starting point for understanding connectivity in undirected graphs is to consider the connected components which can be found in linear time. In directed graphs the starting point is a bit more involved. We need to consider the strong connected components and the meta-graph, which is a DAG, obtained by shrinking each strong connected component into a single node and removing all parallel edges. Obtaining this representation in linear time requires much more work but we can assume that algorithm as a black box. Several basic question on directed graph connectivity rely on the meta-graph and its properties.

1. There are n light bulbs in a garden. These bulbs can be turned on manually by flipping on the switches at the light posts. Also, each light post can broadcast turn-on signals to some other pre-defined light posts in the garden, turning them on. When a light post is turned on, it will automatically broadcast a turn-on signal to its pre-defined light posts.

This signal broadcasting is directional. If a broadcasts to b , it is not necessarily true that b also broadcasts to a .

So one can manually flip on some of the switches to the light posts, and those light posts will broadcast a turn-on signal to other light posts. These will in turn be switched on and broadcast signals to their own pre-defined set of light posts, and so on.

Given each light post in the garden and the respective light posts to which they broadcast, derive a linear time algorithm for finding the minimum number of switches needed to be flipped to light up the whole garden. (Linear time means $O(n + m)$ where n is the number of light posts and m is the number of broadcast associations between them).

Source: *ACM ICPC 2010 World Finals Warmup 2*

Example Case: Number of lights : 5 , Number of broadcast associations: 4

Associations : $1 \Rightarrow 2$, $1 \Rightarrow 3$ $3 \Rightarrow 4$, $5 \Rightarrow 3$

Answer: Minimum number of flips required : 2 ,

Turning on switches 1 and 5 should light up the whole garden

Hints:

- (a) Model the problem using directed graphs.
 - (b) What is the solution if the graph in question is strongly connected?
 - (c) What is the solution if the graph in question is a DAG?
 - (d) What is the solution in general?
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2. Let G be a directed acyclic graph. Prove that G has a unique topological sort if and only if it has a Hamiltonian Path.