## Algorithms \& Models of Computation

## CS/ECE 374, Fall 2020

### 19.6.2

Interval Scheduling: Earliest finish time

## Earliest Finish Time

Process jobs in the order of their finishing times, beginning with those that finish earliest.


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## Optimal Greedy Algorithm

```
R}\mathrm{ is the set of all requests
X\leftarrow\emptyset (* X stores the jobs that will be scheduled *)
while R is not empty
    choose i\inR such that finishing time of i is smallest
        add i to }\boldsymbol{X
        remove from R all requests that overlap with i
return X
```


## Theorem 19.2.

The greedy algorithm that picks jobs in the order of their finishing times is optimal.

## Implementation and Running Time

```
Initially R}\mathrm{ is the set of all requests
X}\leftarrow\emptyset(* X stores the jobs that will be scheduled *
while R}\mathrm{ is not empty
    choose i\inR such that finishing time of i is least
    if i does not overlap with requests in }\boldsymbol{X
        add i to X
    remove i from R
return the set }
```

- Presort all requests based on finishing time. $O(n \log n)$ time
- Now choosing least finishing time is $\boldsymbol{O}(1)$
- Keep track of the finishing time of the last request added to $\boldsymbol{A}$. Then check if starting time of $\boldsymbol{i}$ later than that
- Thus, checking non-overlapping is $O(1)$
- Total time $O(n \log n+n)=O(n \log n)$


## Comments

(1) Interesting Exercise: smallest interval first picks at least half the optimum number of intervals.
(2) All requests need not be known at the beginning. Such online algorithms are a subject of research

## Weighted Interval Scheduling

Suppose we are given $\boldsymbol{n}$ jobs. Each job $\boldsymbol{i}$ has a start time $\boldsymbol{s}_{\boldsymbol{i}}$, a finish time $\boldsymbol{f}_{\boldsymbol{i}}$, and a weight $\boldsymbol{w}_{\boldsymbol{i}}$. We would like to find a set $\boldsymbol{S}$ of compatible jobs whose total weight is maximized. Which of the following greedy algorithms finds the optimum schedule?
a Earliest start time first.
a Earliest finish time fist.
(1) Highest weight first.
d None of the above.
a IDK.

Weighted problem can be solved via dynamic programming. See notes.

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## THE END

(for now)

