1 Suppose you are given a magic black box that somehow answers the following decision problem in polynomial time:

- Input: A CNF formula $\varphi$ with $n$ variables $x_{1}, x_{2}, \ldots, x_{n}$.
- Output: True if there is an assignment of True or False to each variable that satisfies $\varphi$.

Using this black box as a subroutine, describe an algorithm that solves the following related search problem in polynomial time:

- Input: A CNF formula $\varphi$ with $n$ variables $x_{1}, \ldots, x_{n}$.
- Output: A truth assignment to the variables that satisfies $\varphi$, or None if there is no satisfying assignment.
(Hint: You can use the magic box more than once.)
2 An independent set in a graph $G$ is a subset $S$ of the vertices of $G$, such that no two vertices in $S$ are connected by an edge in $G$. Suppose you are given a magic black box that somehow answers the following decision problem in polynomial time:
- Input: An undirected graph $G$ and an integer $k$.
- Output: True if $G$ has an independent set of size $k$, and False otherwise.
2.A. Using this black box as a subroutine, describe algorithms that solves the following optimization problem in polynomial time:
- Input: An undirected graph $G$.
- Output: The size of the largest independent set in $G$.
(Hint: You have seen this problem before.)
2.B. Using this black box as a subroutine, describe algorithms that solves the following search problem in polynomial time:
- Input: An undirected graph $G$.
- Output: An independent set in $G$ of maximum size.


## To think about later:

3 Formally, a proper coloring of a graph $G=(V, E)$ is a function $c: V \rightarrow\{1,2, \ldots, k\}$, for some integer $k$, such that $c(u) \neq c(v)$ for all $u v \in E$. Less formally, a valid coloring assigns each vertex of $G$ a color, such that every edge in $G$ has endpoints with different colors. The chromatic number of a graph is the minimum number of colors in a proper coloring of $G$.
Suppose you are given a magic black box that somehow answers the following decision problem in polynomial time:

- Input: An undirected graph $G$ and an integer $k$.
- Output: True if $G$ has a proper coloring with $k$ colors, and False otherwise.

Using this black box as a subroutine, describe an algorithm that solves the following coloring problem in polynomial time:

- Input: An undirected graph $G$.
- Output: A valid coloring of $G$ using the minimum possible number of colors.
(Hint: You can use the magic box more than once. The input to the magic box is a graph and only a graph, meaning only vertices and edges.)

