# HW 9

Due on **Wednesday, November 2, 2022 at 9am CST**

**CS/ECE 374A: Intro. Algorithms & Models of Computation, Fall 2022** Version: **1.0**

**Submission instructions as in previous homeworks.**

**17** (100 PTS.) Shortest legible walk.

You are given a directed graph $\mathsf{G} = (\mathsf{V}, \mathsf{E})$ with $n$ vertices and $m \geq n$ edges, and a pair of vertices $s$ and $t$. In addition, every edge $e \in \mathsf{E}$, has a *weight* $w(e) > 0$, and an associated character $c(e) \in \Sigma$. Thus, a walk $\pi = e_1 e_2 e_3 \ldots e_k$ encodes the string $c(\pi) = c(e_1)c(e_2)\cdots c(e_k)$.

You are also given a DFA $M = (Q, \Sigma, \delta, q_0, A)$, where $N = |Q|$, and $|\Sigma| = O(1)$.

Describe **formally** an algorithm, as fast as possible (the running time depends on $n, m$ and $N$), that computes the shortest walk $\pi$ from $s$ to $t$, such that $c(\pi) \in L(M)$. Formally, let

$$\Pi = \{c(\pi) \in L(M) \mid \pi \text{ is a walk in } \mathsf{G} \text{ from } s \text{ to } t\}.$$

The task is to compute $\arg\min_{\pi \in \Pi} w(\pi)$.

**18** (100 PTS.) Paths and cycles.

Let $\mathsf{G}$ be a directed graph with $n$ vertices and $m$ edges, with positive numbers as weights on the edges.

**18.A.** (50 PTS.) You are given in addition to $\mathsf{G}$, two vertices $s, t \in \mathsf{V}(\mathsf{G})$, and a parameter $k$. Describe an algorithm, as fast as possible, that computes the shortest path in $\mathsf{G}$ between $s$ and $t$, where you are allowed to travel on $k$ edges for free. What is the running time of your algorithm?

**18.B.** (50 PTS.) The ***heft*** of a cycle $C$ in $\mathsf{G}$, is the minimum weight edge in $C$. Describe an algorithm, as fast as possible, that computes the maximum heft cycle in $\mathsf{G}$.