3 (100 pts.) 374 Balanced.

A string $s$ over $\Sigma = \{0, 1\}$ is balanced if (i) $\#_0(s) = \#_1(s)$, and for any prefix $p$ of $s$ we have that $\#_0(p) \geq \#_1(p)$. Here, for any character $c \in \Sigma$, and any string $w \in \Sigma^*$, the quantity $\#_c(w)$ is the number of times the character $c$ appears in $w$. Thus, the strings

$$01010101, \quad 00101100101011, \quad 00011101, \quad \text{and} \quad 010011,$$

are balanced, while

$$10, \quad 001, \quad 001110, \quad 000111011111, \quad \text{and} \quad 01001110,$$

are not balanced. A string $w$ is 374 balanced if $w$ is balanced, and for any prefix $p$ of $w$, we have that $0 \leq \#_0(p) - \#_1(p) \leq 374$.

For both languages specified below, describe formally a DFA that accepts them. In addition, explain informally and precisely the idea beyond your DFA and how it works.

3.A. (50 pts.) Let $L_1$ be the language of all 374 balanced strings.

3.B. (50 pts.) Let $L_2$ be the language of all binary strings $w$, such that:

(i) $w$ is 374 balanced,

(ii) $|w|$ is divisible by 16, and

(iii) $w$ contains 0000 as a substring.

(The language of all balanced strings is not regular, so this question is interesting because the more restricted languages $L_1$ and $L_2$ are regular.)

4 (100 pts.) Freedom of regular expressions.

For each of the following languages over the alphabet $\{0, 1\}$, give a regular expression that describes that language, and briefly argue why your expression is correct.

4.A. (30 pts.) The language containing all strings that do not contain 000 as a substring.

4.B. (70 pts.) All strings that do not contain 0110 as a subsequence.

(Hint: (A) Break the input string into runs – a run of a string $w$ is a maximal substring $s$ all made of the same character. (B) You might want to solve an easier version of this question first, where 010 is a forbidden subsequence.)