374 12/1/2022

27: more NPC problems

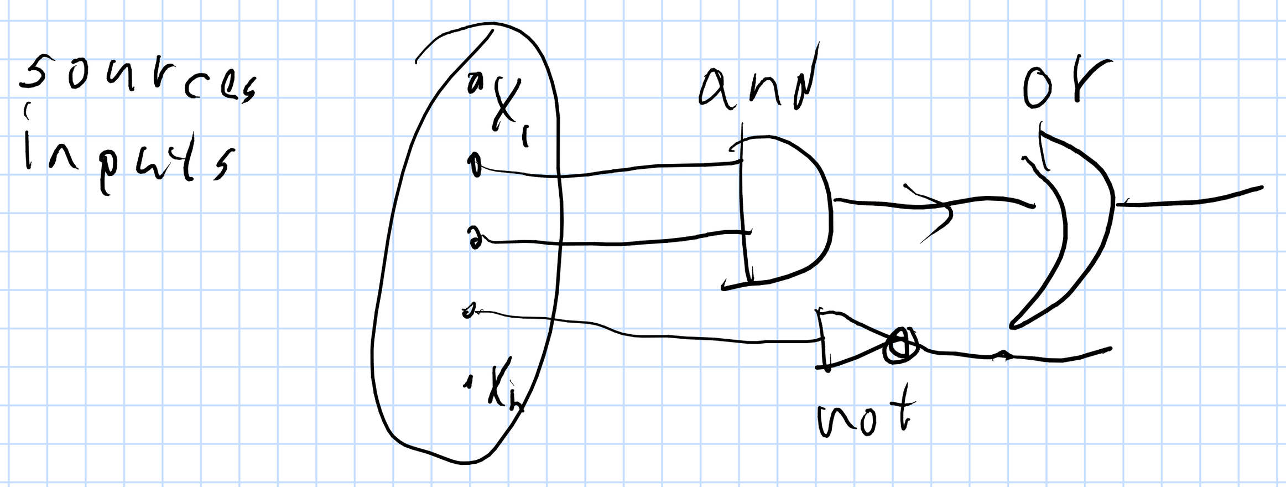
Cook Levin theorem

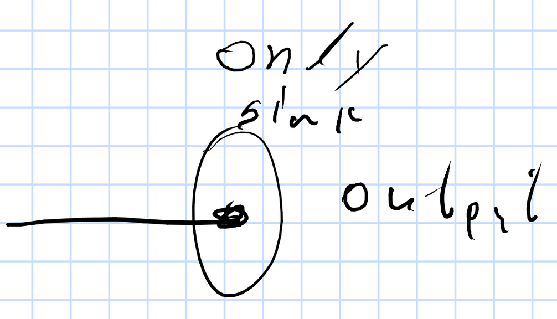
CSAT

Input: Boolean circuit with n inputs and one output.

Q: Is there an input to the circuit that the output is 1/TRUE.

Circuit ≡ DAG



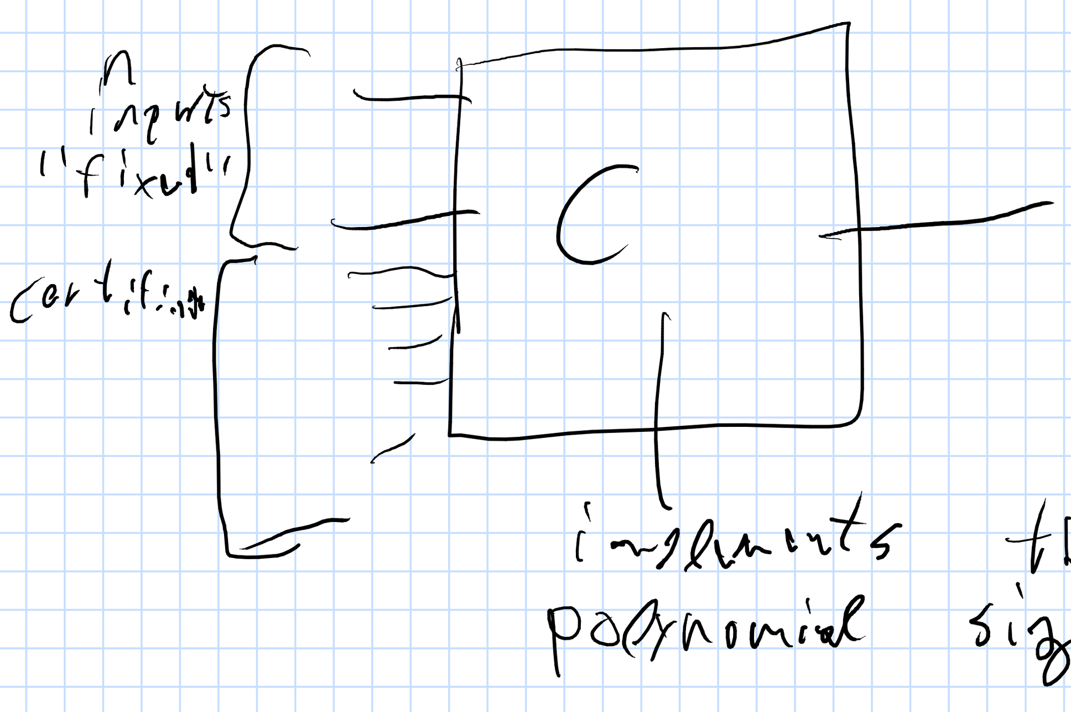


A problem Π is in NP if ∃ a poly time verifier for it.

⟹ verifier_Π(instance, certificate) runs in poly time

⟹ TM that implement the verifiers and runs in poly time

⟹ "compile TM to a circuit (Cook-Levin Th)



implements the certifier for Π.
polynomial size

CSAT is NPC. (Cook-Levin)

Claim: SAT is NPC.

proof

- SAT is in NP. ✓
- need to show reduction from CSAT to SAT.

$CSAT \leq_p SAT$.

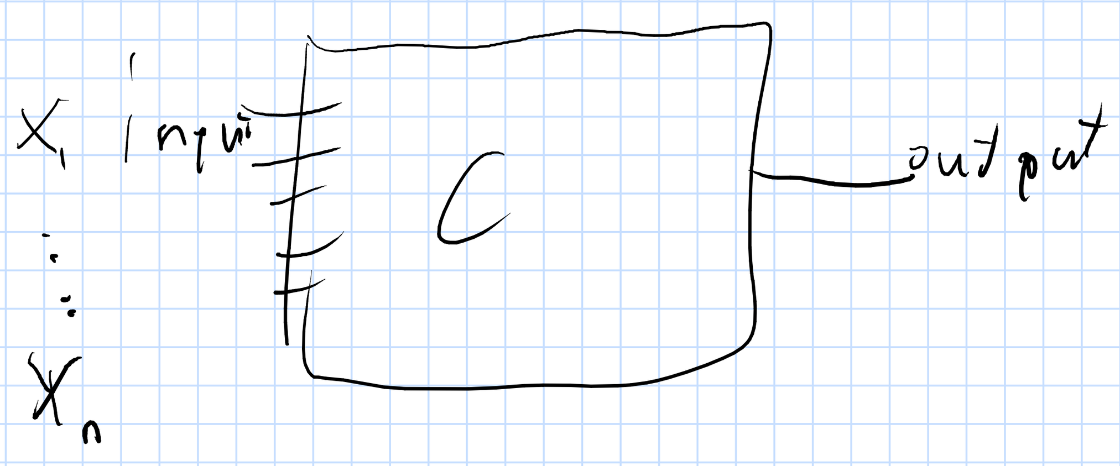
SAT
Input: CNF formula
Q: Is there a satisfying assignment

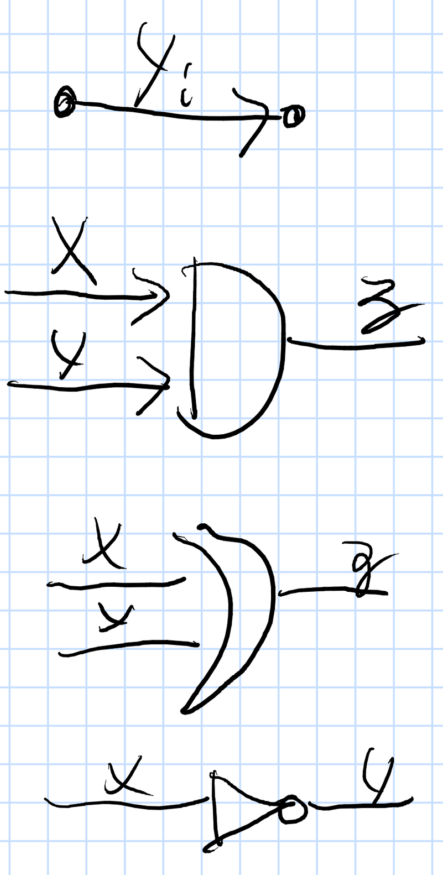
CNF formula

$\bigwedge_{j=1}^{m} C_j$

$C_j = \bigvee_{i=1}^{k} \tilde{x}_i$

input is a circuit C





$z = x \wedge y$

$z = x \vee y$

$y = \bar{x}$

V_{output}

x	y	z	$z = x \wedge y$	$x \vee y \vee \bar{z}$	$x \vee \bar{y} \vee \bar{z}$
0	0	0	1	1	1
0	0	1	0	0	1
0	1	0	1	1	1
0	1	1	0	1	0
1	0	0	1	1	
1	0	1	0	1	
1	1	0	0	1	
1	1	1	1	1	

$CSAT =_p SAT =_p 3SAT =_p IS =_p CLIQUE =_p VC$
$=_p$ HAMILTONIAN CYCLE/PATH

Graph coloring ⟨G, k⟩

Q: Can the graph be colored with k colors.

3COLOR: ⟨G⟩

Q: Is G three colorable?

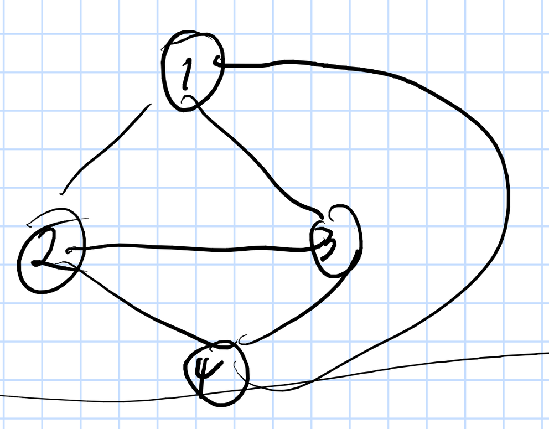
- 3COLOR is in NP.
- $3SAT \leq_p 3COLOR$.

⟹ 3COLOR is NP-Complete.

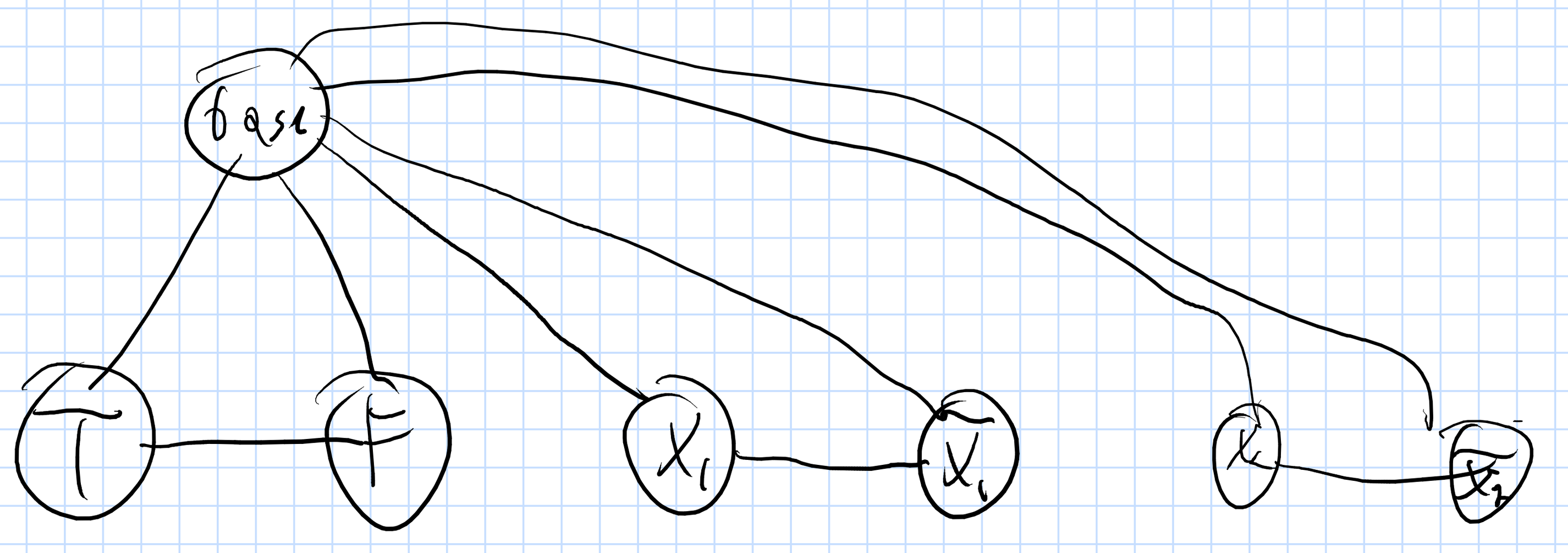
$O(n^{1/4})$ colors

F is a 3SAT formula

$x_1, x_2, \ldots, x_n$



K_4



$(x \vee y \vee z)$

subset sum

$i_1, i_2, \ldots, i_n$ integer positive numbers

t: target number

Q: Is there a subset of the numbers that add up to t.?

subset sum is NPC

Exercise

If all the n numbers in subset sum are at most $O(n^3)$ than the problem can be solved in poly time?

PARTITION

$\{i_1, i_2 \ldots i_n\} = I$

$\exists S \subseteq I \quad \sum_{s \in S} s = \sum_{s \in I \setminus S} s$?