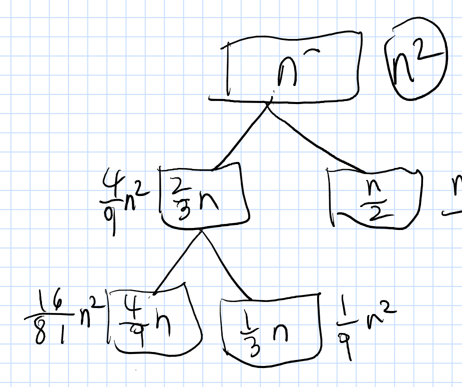
374 Lecture 22 11/3/22
midterm review

1. $T(n) = T(\frac{2}{3}n) + T(\frac{n}{2}) + O(n^2) = O(n^2) \le cn^2$



n^2

$= \left(\frac{4}{9} + \frac{1}{4}\right)n^2 < \frac{2}{3}n^2$

$\boxed{\sum_{i=0}^{\infty} \left(\frac{2}{3}\right)^i n^2 = O(n^2)}$

$T(n) \le T(\frac{2}{3}n) + T(\frac{n}{2}) + \alpha n^2$

$\boxed{T(k) \le ck^2 \quad k<n}$

Induction step:

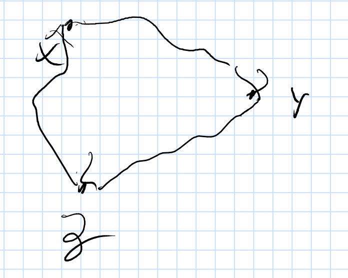
$T(n) \le \left(\frac{2}{3}n\right)^2 \cdot c + \left(\frac{1}{2}n\right)^2 \cdot c + \alpha n^2$

$\le \underbrace{\left(\frac{4}{9} + \frac{1}{2} + \frac{\alpha}{c}\right)}_{<1 \text{ if } c \text{ is large enough}} cn^2 \le cn^2.$

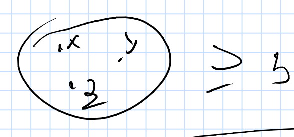
$c = 10\alpha$

1.B.

DFS/BFS $O(n+m)$



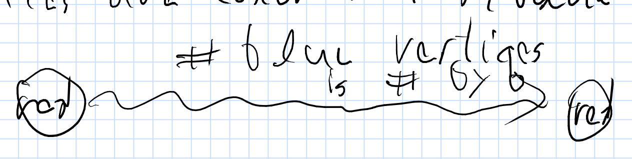
SCC $O(n+m)$ $O(n+m)$

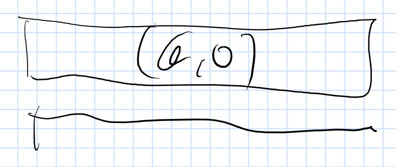


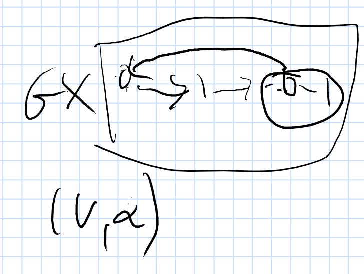
≥ 3

5. G, +ve $w(e)$

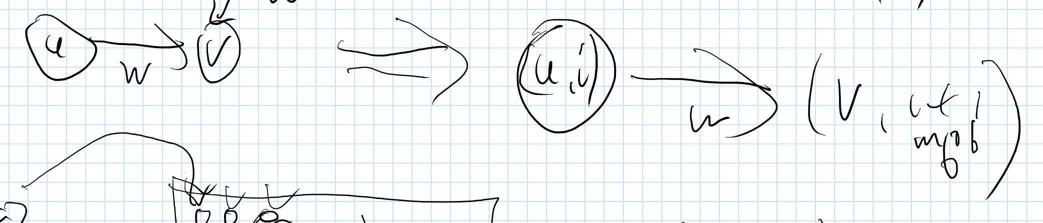
vertices are colored red/blue



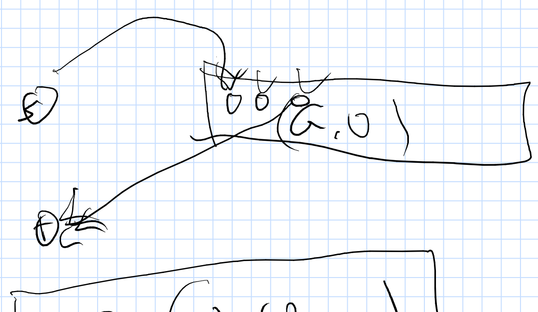




(v, α)



$(v, i+1 \bmod b)$



$O(b(n+m))$

$O(\tilde{n}\tilde{m}) = O(bn \cdot bm)$

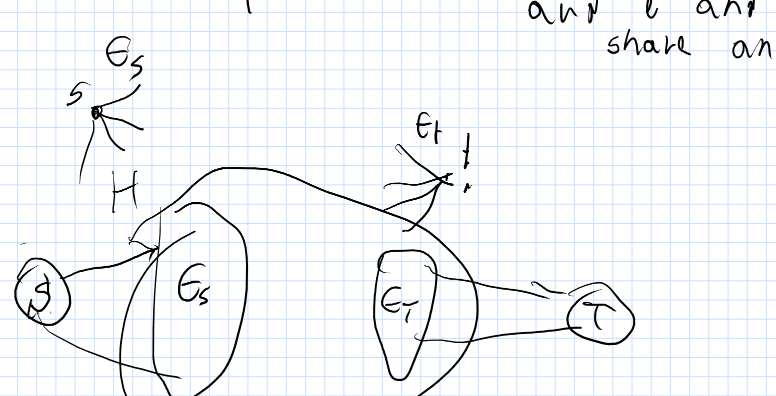
$= O(b^2nm)$

$\boxed{O(n^2b^2nm).}$

$G=(V,E)$ undirected

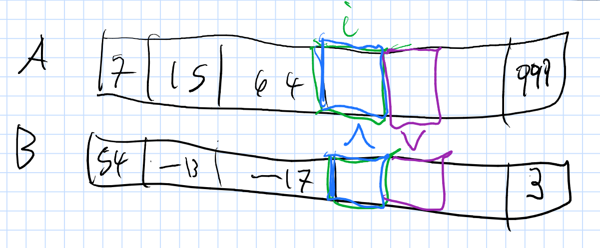
$H = (G, \hat{E})$

$\hat{E} = \{ee' \mid e, e' \in E(G)$ and angle$(e,e') > 90°\}$
and e and e' share an endpoint

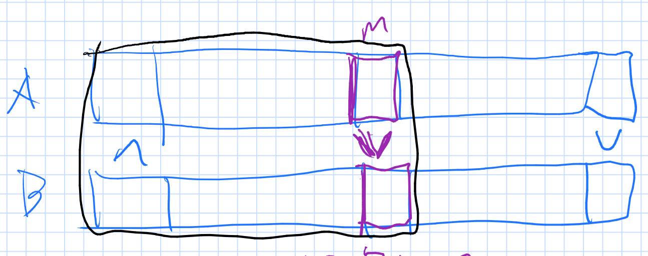


$\frac{m}{n}$ n

$\sum_{v \in V(G)} \binom{d(v)}{2} \le \sum_{v \in V(G)} (d(v))^2 \le O(mn)$
$O(n^3)$

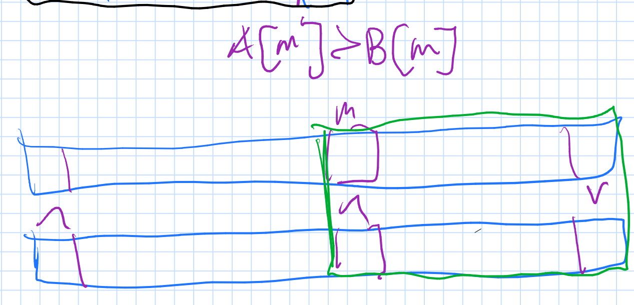


$O(n)$



$n=2$

$A[m] > B[m]$



$Q(n) = O(1) + Q(\lceil \frac{n}{2} \rceil + 1)$

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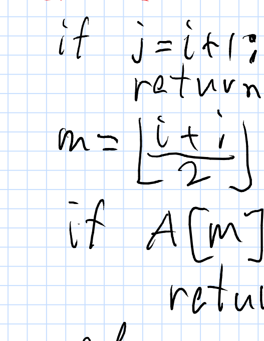
$\boxed{Q(n) = O(1) \quad n \le 4}$

$Q'(n) = O(1) + Q(\frac{2}{3}n)$

$Q(n) = O(Q'(n)) = O(\log n)$




```
find_location(A[i...j], B[i...j]):
    if j=i+1:
        return i.
    m = ⌊(i+j)/2⌋
    if A[m]<B[m]:
        return find_location(A[m...j], B[m...j])
    else
        return find_location(A[i...m], B[i...m])
```

correctness: The alg maintains the invariant that in a recursive call on range $i \ldots j$, we have the property than

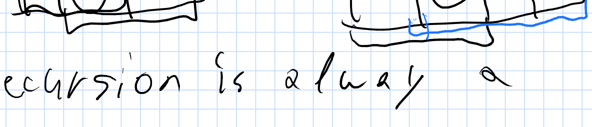
$A[i] < B[i]$

and $A[j] > B[j]$

as such when the recursion reach a subproblem of size 2, it found the desired location.

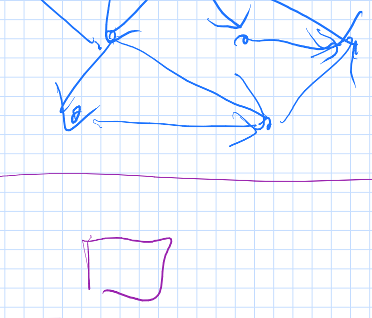
Importantly, if the subproblem is of size $\alpha > 2$ the the recursive call is on a subproblem of size

$\ge \lceil \frac{\alpha}{2} \rceil \ge 2.$



namely the bottom of the recursion is alway a problem of size 2.

n boxes
m locks



$O(n+m)$ BFS.

G^{SCC}