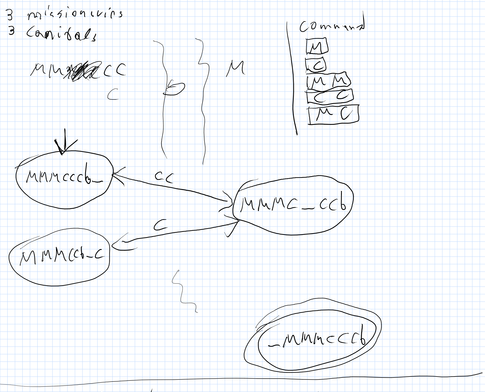
324 Lecture 17 10/18/22

Graphs

Canibals and missionaries problem

3 missionaries
3 canibals



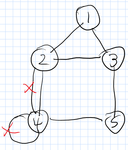
V: set of vertices $\quad n=|V|$ vertices

$V=[n]=\{1,2,3,\dots,n\}$

Edge: $\{u,v\} \equiv$ edge between u and v (undirected edge)

$uv=\{u,v\}$

$E=\{$set of all edges in the graph$\}$

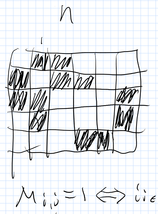


$V=\{1,2,3,4,5\}$

$E=\{\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{3,5\}\}$
$=\{12,13,23,24,45,35\}$

$m=|E|$ number of edges.

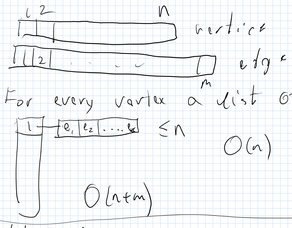
simple graph: Graph with no self loops or parallel edges.



Adjacency matrix

$M_{ij}=1 \iff ij\in E(G)$
0 otherwise

Adjacency lists



For every vertex a list of its neighbors:

$\le n$

$O(n)$

$O(n+m)$

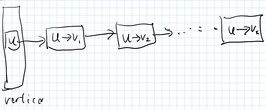
directed graphs

$u\to v$, (u,v): directed edge from u to v.

$(u\to v)$

V E ← edges $\quad n=|V| \quad m=|E|$

vertices



Adjacency matrix

$M_{i,j}=1 \iff (i\to j)\in E(G)$.

Graph search/exploration

$u,v\in V$ are connected in G (undirected graph if there is a path from u to v in G.

Path is a sequence of distinct vertices $v_1,v_2,v_3,\dots v_k$ s.t. $v_i v_{i+1}\in E(G)$ $\forall i$.

u and v are connected if $\exists$ path $u=v_1,\dots,v_k=v$ in G.

Walk is a path that allowed to contain a vertex several times.

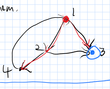
a path in directed graph $v_1,v_2\dots v_k$ is a sequence of distinct vertices s.t. $(v_i\to v_{i+1})\in E(G)$

v is reachable from u if there exists a path from u to v in the graph.

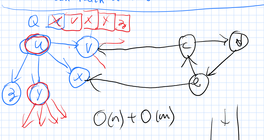
1. Is u and v connected? (undir)
2. What is the connected component of u?

Undir. The connected component of a vertex u is all the vertices that are connected to u via a path.

reachable set $\mathrm{rch}(u)$ = set of all vertices that have a path from u that reaches them.



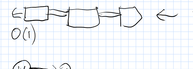
$S\subseteq V(G)$, G is directed, is strongly connected if for any $u,v\in V(S)$ we have
u can reach v in G
v can reach u in G



$O(n)+O(m)$

DFS
Depth first search
] Q is a stack

BFS
Breadth first search
] Q is a FIFO queue
First in first out



$O(1)$

G: directed graph
u: vertex
$\mathrm{scc}(u)$
$\mathrm{rch}(u)$ = all vertices that u reaches
$\mathrm{rch}(u)$ = all the vertices that a path from them reaching u.

G^{rev} = G after reversing all the edges
$O(n+m)$

$\mathrm{rch}(u,G)=\mathrm{rch}(u,G^{rev})$ $\quad O(n+m)$

$\mathrm{scc}(u)=\mathrm{rch}(u)\cap\mathrm{rch}(u)$.