

374 Lecture 8

9/15/22

More on context Free Grammars

$$G = (V, T, P, \underline{s})$$

V : set of variables (non-terminals)

T : set of terminals (characters in Σ)

P : production rules

$$\boxed{\text{var}} \xrightarrow{\downarrow} (V \cup T)^*$$

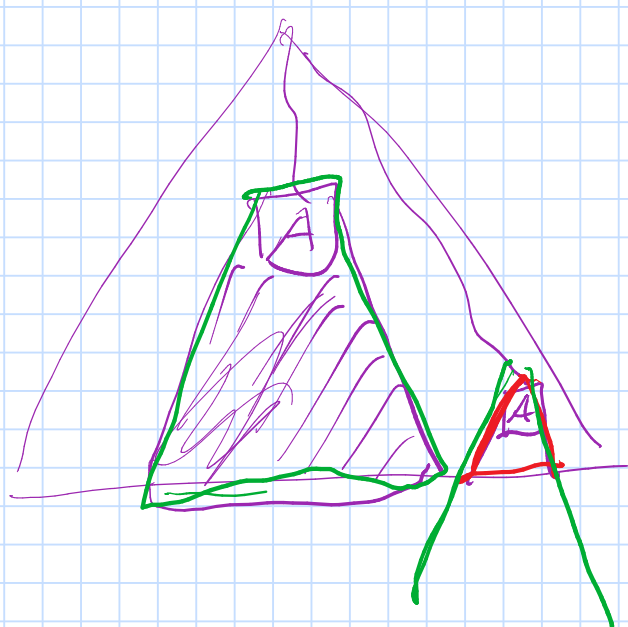
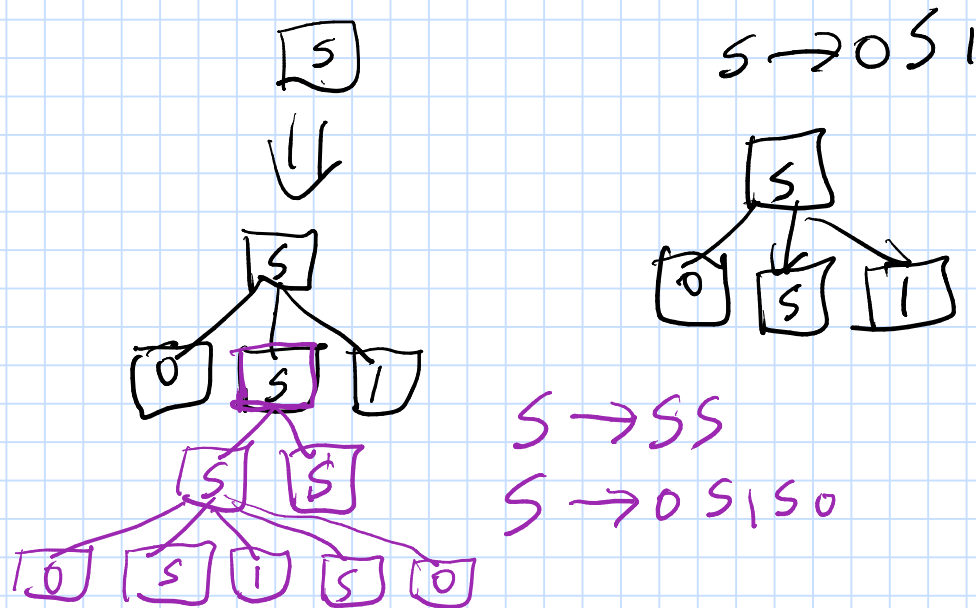
$$\boxed{S \rightarrow \epsilon \mid 0S1} \quad \{0^n 1^n \mid n \geq 0\}$$

$$S \rightarrow \boxed{0S1} \rightarrow 00S11 \rightarrow 00\epsilon 11 = \boxed{0011}$$

$\downarrow$
 $S \rightarrow \epsilon$

$$S \rightarrow T \mid \epsilon$$

$$T \rightarrow S$$



$$xyzvw = a^i b^j c^i \in L = \{a^n b^n c^n \mid n \geq 0\}$$

$$xy^2zv^2w \in L \quad |y| + |v| \geq 0$$

Case analysis:

$$- y = a^s b^t \quad s \geq 0 \quad t \geq 0$$

$$xy^2zv^2w = \boxed{a \dots a} \boxed{aaabbb} \boxed{aaaabb} \dots$$

y

$xy^2zv^2w \notin L$. contradiction.

$$- v = b^s c^t$$

$$xy^2zv^2w = \dots \boxed{bbbccccc} \boxed{bbkkcc} \dots$$

$\downarrow$
contradiction

$$- y = b^s c^t \quad \text{impossible}$$

$$- y = a^s b^t \quad \text{"}$$

$$- y = a^s b^t c^u \quad \text{"}$$

$$- v = b^s c^t \quad \text{"}$$

$$v = a^s b^t$$

$$v = b^s c^t$$

$$xy^2zv^2w$$

$$xy^2zv^2w$$

$$y = a^s \quad v = b^t$$

$$xy^2zv^2w = a^{i+s} b^{i+t} c^i \quad s+t \geq 0$$

$\downarrow$
contradiction

CFG closed under:

- concat

$$S_1 \quad S_2$$

$$S \rightarrow S_1 S_2$$

$$S \rightarrow \epsilon \mid S \mid SS \quad \text{Kleene star}$$

- Under union

$$S \rightarrow S_1 \mid S_2$$

CFL are NOT closed under complement

Theorem

L is regular

L' is CFL

$\Rightarrow L \cap L'$ is CFL.

CFL not closed under intersection

$$\{a^i b^j c^k \mid i, j \geq 0\} \cap \{a^i b^j c^k \mid i, j \geq 0\}$$

$$3 + 6 + 9$$

