374 Lecture 7 CFGs and PDAs

9/13/22

Context free grammars

PDA = push down automatas

$\boxed{\text{computers}} \to \boxed{\text{Turing machine}}$ = DFA + tape

$\to \boxed{\text{PDA}}$ = DFA + stack

$\boxed{\text{DFA/NFA/RG}}$

$\boxed{S \to \varepsilon \mid 0S1}$ (variables,

$S \to \varepsilon$

$S \xrightarrow{1} 0S1 \xrightarrow{2} 00S11 \xrightarrow{3} 00\varepsilon11 = 0011$

$S \rightsquigarrow 0011$

$S \rightsquigarrow_3 0011$

$L(S) = \{w \in \Sigma^* \mid S \rightsquigarrow w\}$

$= \{0^n 1^n \mid n \geq 0\}$

G: $S \to \varepsilon \mid A$

$A \to AA \mid \varepsilon \mid \text{'('}A\text{')'}$

$L(G) = \{\varepsilon, (), (()), (())(), \dots\}$

$S \to A \to AA \to (A)A \to ((A))A \to ((A))(A) \to (())(A) \to (())()$

$L(G)$ = balanced parenthesis strings.

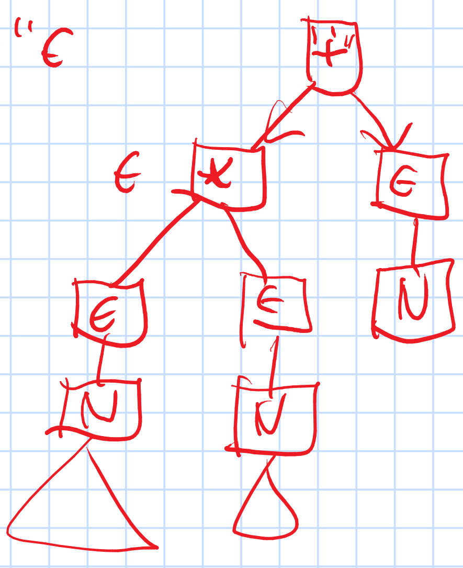
$D \to 0 \mid 1 \mid 2 \dots \mid 9$

$N \to D \mid DN$

$E \to N \mid E\text{"+"}E \mid E\text{"*"}E \mid (E) \mid E\text{"-"}E \mid E\text{"/"}E$

E: start variable

$E \to E\text{"+"}E$



3*7+5

$L = \{0^n 1^m \mid m \geq n\}$

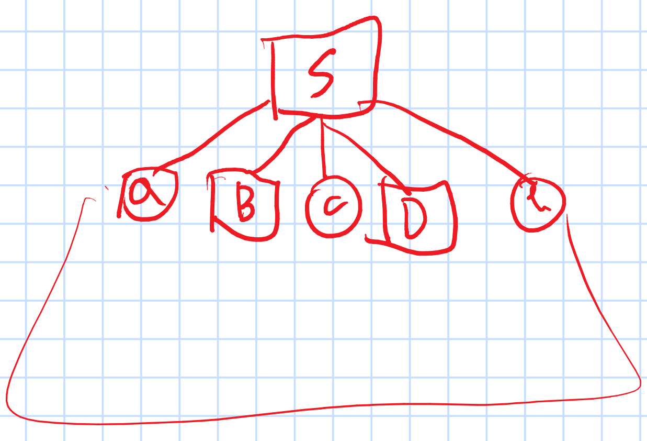
$0^n 1^m = \underbrace{0^n 1^n} \underbrace{1^{m-n}}$

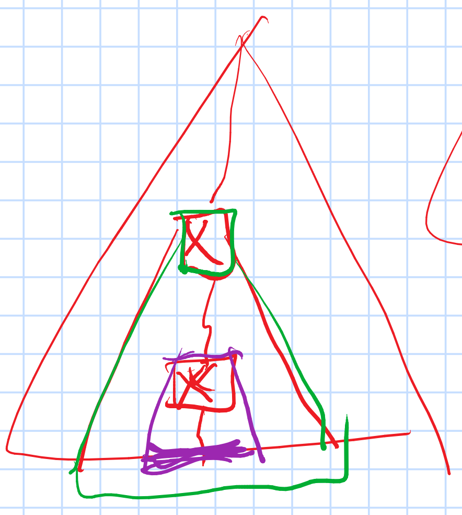
$\boxed{A \to \varepsilon \mid 0A1 \quad B \to \varepsilon \mid 1B \quad \underline{S} \to AB}$

$A \to aA \to aaB \to aabC \to$

$L = \{0^n 1^n 2^n \mid n \geq 0$ is not regular.

$S \to aBcDe$





$L_1 = \{0^n 1^n 2^j \mid n, j \geq 0\}$ CFL

$L_2 = \{0^j 1^n 2^n \mid n, j \geq 0\}$ CFL

$L_1 \cap L_2 = \{0^n 1^n 2^n \mid n \geq 0\}$ not CFL

FL not closed under complement

$L = \{0^i 1^j 2^k \mid \underbrace{i \neq j} \text{ or } j \neq k\} \leftarrow$ CFL

"$\bar{L}$" $= \{0^i j$

$L_1 = \{0^i 1^j \underbrace{2^k} \mid i \neq j\} = L(S)$

$S \to AT \mid BT \qquad T \to 2T \mid \varepsilon$

$A \to 0A \mid 0D \qquad B \to B1 \mid D1$

$D \to 0D1 \mid \varepsilon$

$L_1 \quad L_2$

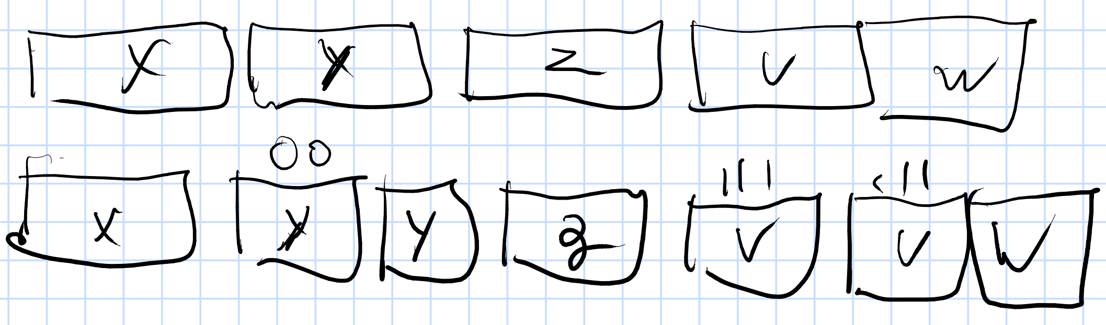
$G_1 \quad G_2$

$S_1 \quad S_2$

$\boxed{S \to S_1 \mid S_2}$

$X \to \underbrace{aYbZcdeFaA}_{(V \cup T)^*}$

$00 \cdots 01 \cdots 12 \cdots 2$



$0^n 1^n 2^n$

$0^{n+t} 1^{n+s} 2^n$