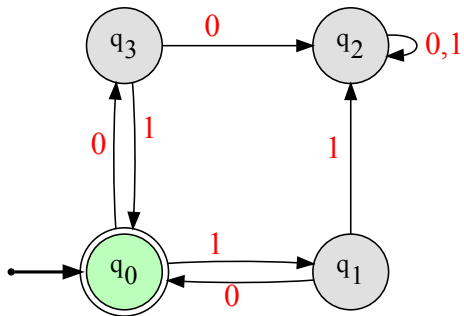


# Deterministic Finite Automata (DFAs): Examples

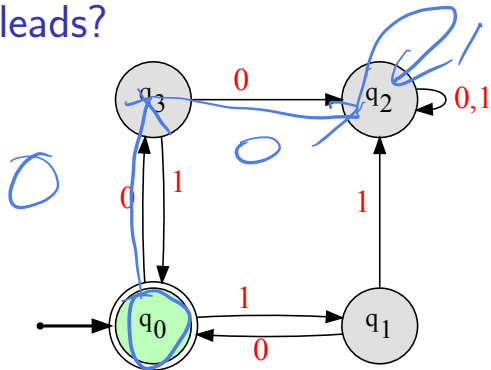
## Lecture 3

Tuesday, August 30, 2022

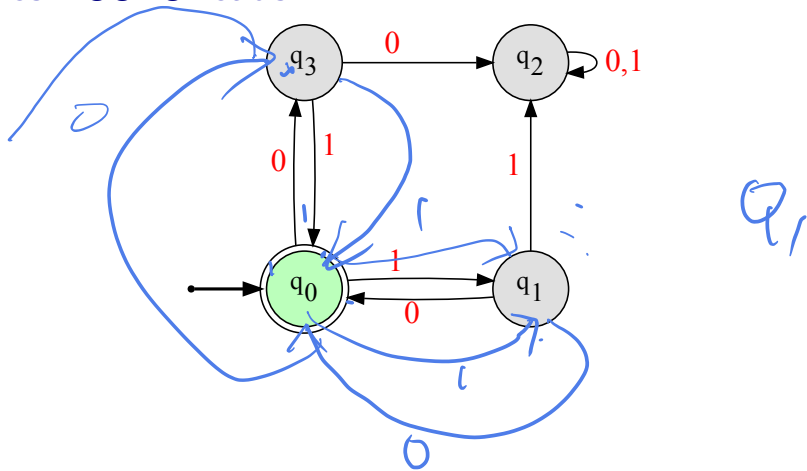
# Graphical Representation/State Machine



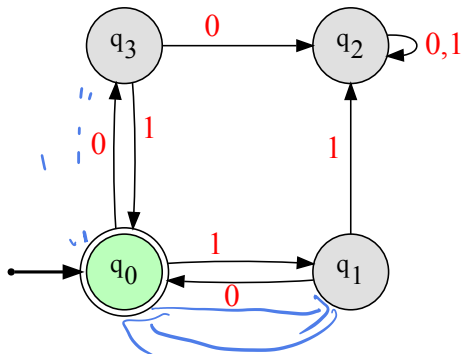
Where does **001** leads?



Where does **10010** leads?



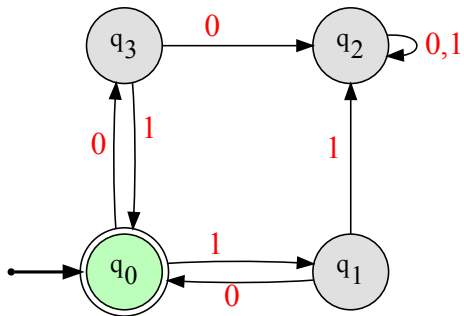
Which strings end up in an accepting state?



$\Sigma$   
01  
0101  
 $(01)^*$   
 $(10)^*$

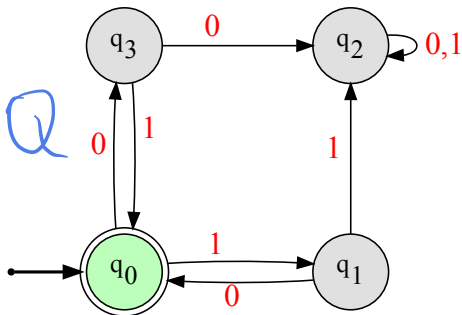
$(01 + 10)^*$

Which strings end up in an accepting state?



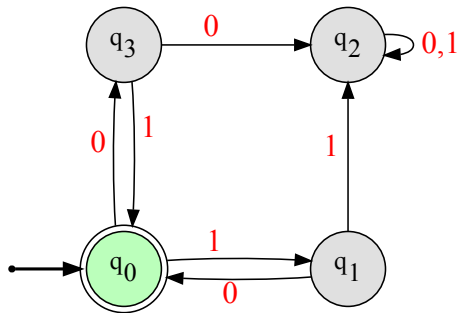
## Example

$$f: Q \times \Sigma \rightarrow Q$$



- ▶  $Q = \{q_0, q_1, q_1, q_3\}$  ~ set of states
  - ▶  $\Sigma = \{0, 1\}$  ~ input alphabet
  - ▶  $\delta$
  - ▶  $s = q_0$
  - ▶  $A = \{q_0\}$  ~ set of accepting states
- $\cap$   
 $Q$

## Example: The transition function



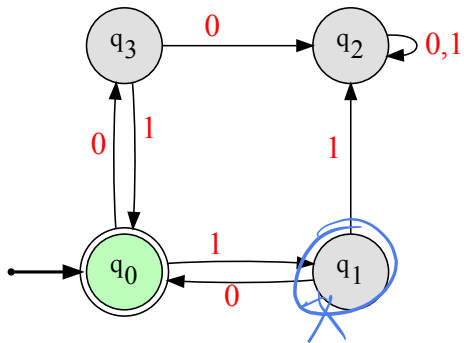
$\delta$  :

state $q \in Q$	input $c \in \Sigma$	result $\delta(q, c) \in Q$
$q_0$	0	$q_3$
$q_0$	1	$q_1$
$q_1$	0	$q_0$
$q_1$	1	$q_2$
$q_2$	0	$q_2$
$q_2$	1	$q_0$
$q_3$	0	$q_2$
$q_3$	1	$q_0$



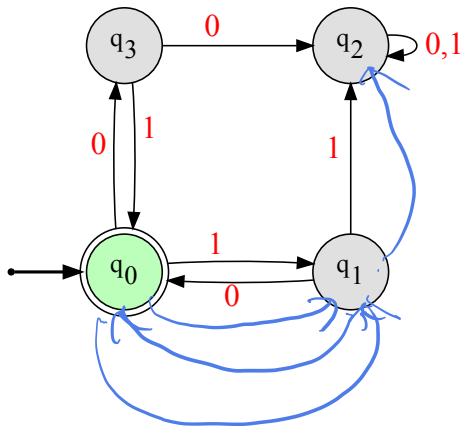
What is  $\delta^*(q_1, \epsilon)$ ?

11  
q<sub>1</sub>



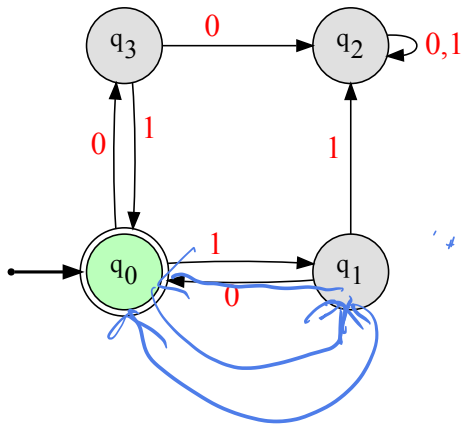
What is  $\delta^*(q_0, 1011)$ ?

11  
 $q_2$

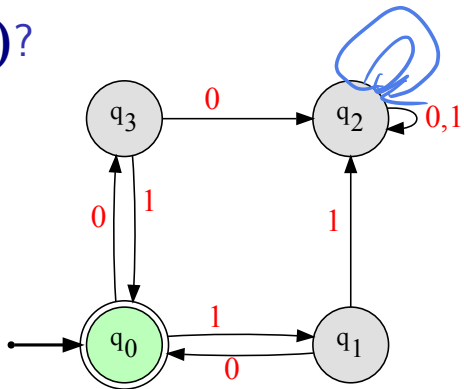


What is  $\delta^*(q_1, 010)$ ?

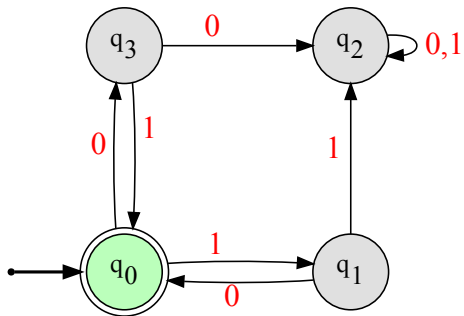
q<sub>0</sub>



What is  $\delta^*(q_2, 10)$ ?

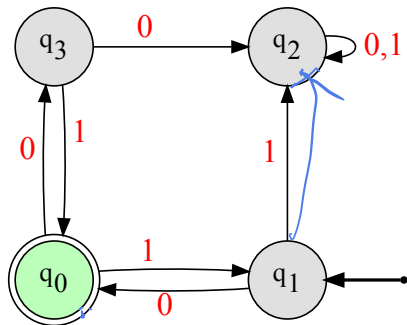


What is  $L(M)$ ?

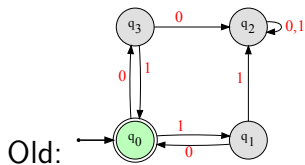


$(01+10)^*$

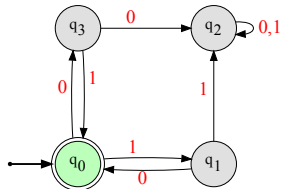
What is  $L(M)$  if start state is changed to  $q_1$ ?



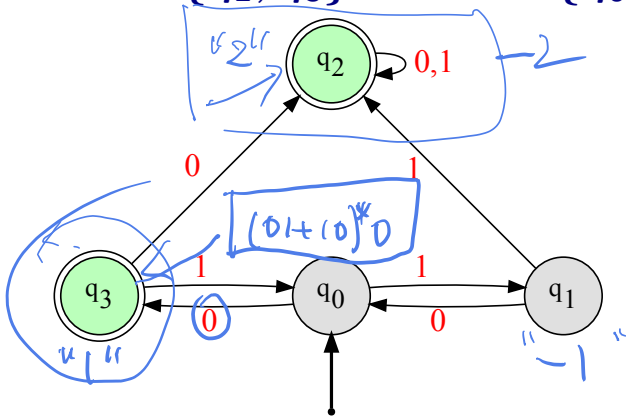
$\emptyset (01+10)^*$



$L(M)$  if final/accept states are  $\{q_2, q_3\}$  instead of  $\{q_0\}$ ?



Old version:

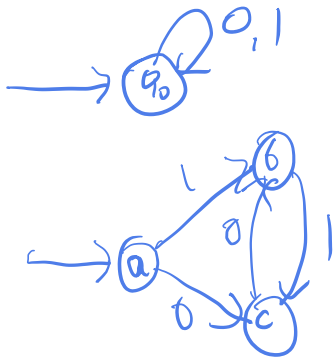


Redraw

$L(q_2) = \text{all binary strings with prefix with balance 2 or -2}$   
 $\#_0(w) - \#_1(w) = 0$

# DFA example: $L = \emptyset$

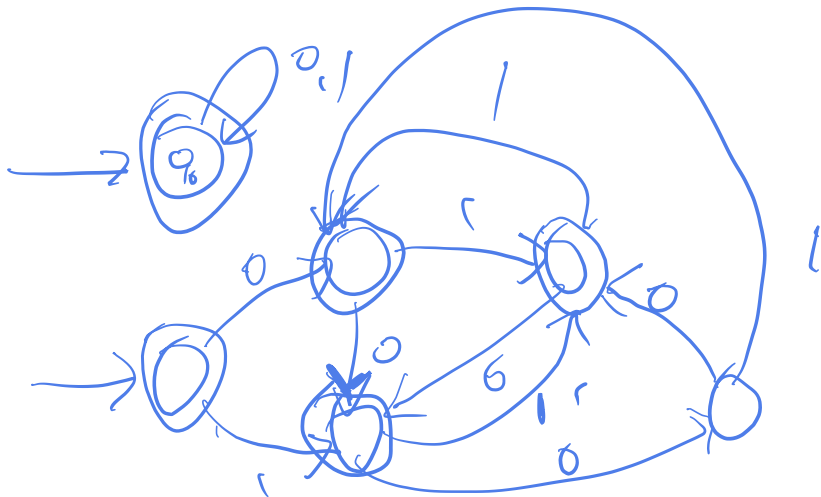
Assume  $\Sigma = \{0, 1\}$ .





# DFA example: $L = \Sigma^*$

Assume  $\Sigma = \{0, 1\}$ .



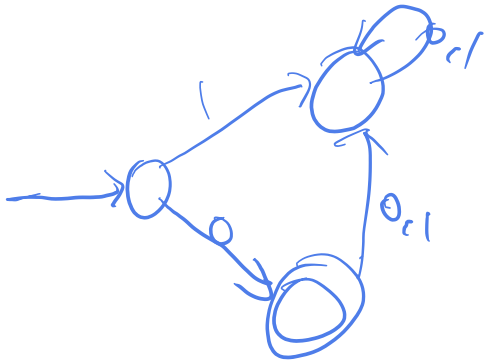
DFA example:  $L = \{\epsilon\}$

Assume  $\Sigma = \{0, 1\}$ .



DFA example:  $L = \{0\}$

Assume  $\Sigma = \{0, 1\}$ .



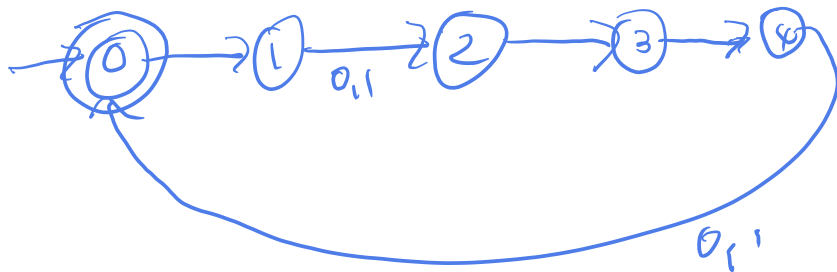
# DFA Construction: Examples

## Example II: Length divisible by 5

Assume  $\Sigma = \{0, 1\}$ .

$L = \{w \in \{0, 1\}^* \mid |w| \text{ is divisible by } 5\}$

4 + 50'

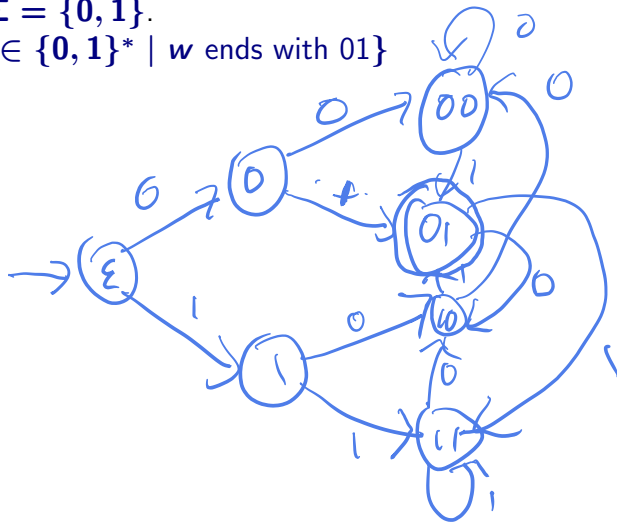


# DFA Construction: examples

## Example III: Ends with 01

Assume  $\Sigma = \{0, 1\}$ .

$L = \{w \in \{0, 1\}^* \mid w \text{ ends with } 01\}$

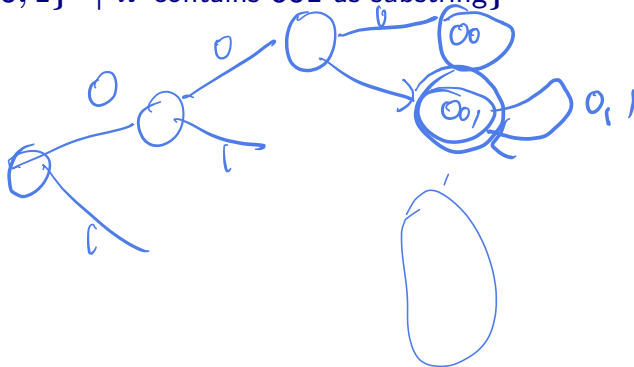


# DFA Construction: examples

## Example IV: Contains 001

Assume  $\Sigma = \{0, 1\}$ .

$L = \{w \in \{0, 1\}^* \mid w \text{ contains } 001 \text{ as substring}\}$

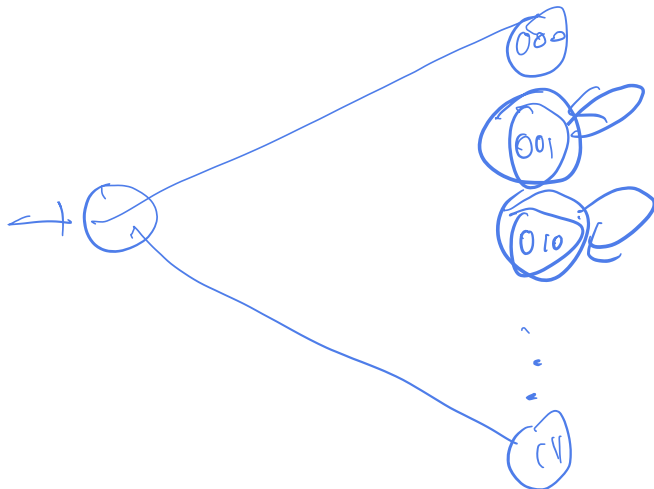


# DFA Construction: examples

Example V: Contains 001 or 010

Assume  $\Sigma = \{0, 1\}$ .

$L = \{w \in \{0, 1\}^* \mid w \text{ contains } 001 \text{ or } 010 \text{ as substring}\}$



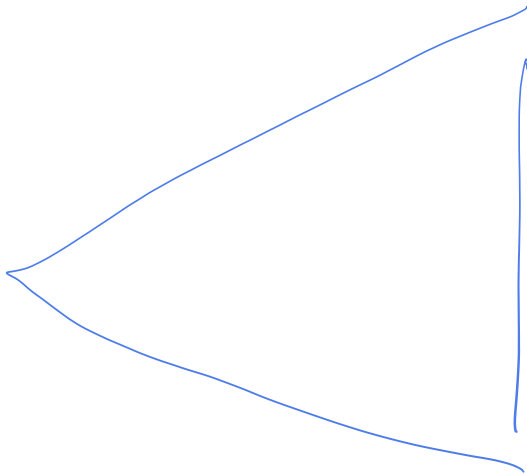
# DFA construction examples

Example VI: Has a **1** exactly **k** positions from end

Assume  $\Sigma = \{0, 1\}$ .

$L = \{w \in \Sigma^* \mid w \text{ has a } \mathbf{1} \text{ located } k \text{ positions from the end of } w\}$ .

*Handwritten:*  $k$  levels





Example:  $L = \{\text{Binary numbers congruent to } 0 \bmod 5\}$