Regular langs 8/25/22

Languages

Language ≡ set of words

word ≡ string

Σ: Alphabet (finite set of characters).

x, y two strings

$xy = x \circ y =$

X, Y sets of strings

$XY = \{x \circ y \mid x \in X \text{ and } y \in Y\}$

$\varepsilon =$ empty string

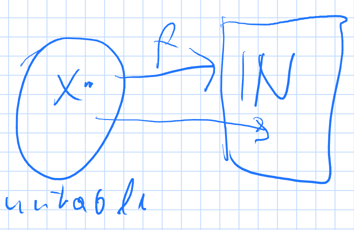
$\varepsilon x = x$ for any string x.

$\Sigma\Sigma = \Sigma^2$, Σ^i

$\Sigma^* = \bigcup_{i=0}^{\infty} \Sigma^i$

$\mathbb{N} = \{1, 2, 3, 4, \dots\}$ natural numbers (countable

A set X is countable if there is injection f from X to $\mathbb{N}$.



infinite countable sets

Cantor proved $\mathbb{N} \neq \mathbb{R}$

$(0,1)$ 0.01011101

f	
x_1	0 0 1 1 0
x_2	0 1 0 1 1 1
x_3	0 0 1 0 1 1
	1 0 1 1 1

Regular lang

$L = \{$all prime numbers written in base 10$\}$

$= \{2, 3, 5, 7, \dots\}$

Regular lang

$L = \{\}$ regular

$L = \{\varepsilon\}$

$L = \{a\}$ $a \in \Sigma$

X, Y are regular

XY is regular

$\Rightarrow L = \{w\}$ $w \in \Sigma^*$ is regular

$w = w_1 w_2 \dots w_n$

$\{w_1\}\{w_2\}\{w_3\} \dots \{w_n\}$

X, Y are regular $\Rightarrow X \cup Y$ is regular

$L = \{s_1, s_2, \dots, s_m\}$

$= \bigcup_{i=1}^{m} \{s_i\}$

L is regular $\Rightarrow L^* = \bigcup_{i=0}^{\infty} L^i$

$L = \{a, ba\}$

$L^* = \{\varepsilon, a, ba, aa, aba, baa, baba, \dots\}$

Complement L is regular $\Rightarrow \overline{L} = \Sigma^* \setminus L$

$= \{w \in \Sigma^* \mid w \notin L\}$

Q: X, Y are regular languages

is $X \cap Y$ regular?

$\overline{X \cap Y} = \overline{X} \cup \overline{Y}$ de Morgan Laws

$\overline{X \cup Y} = \overline{X} \cap \overline{Y}$

$X \cap Y = \overline{(\overline{X} \cup \overline{Y})}$

$X \Rightarrow \overline{X}$ $Y \Rightarrow \overline{Y}$ $\overline{X} \cup \overline{Y}$ is regular

$\overline{\overline{X} \cup \overline{Y}} = X \cap Y$

$L_1, L_2, L_3, L_4 \dots$ infinite sequence of regular languages

$\bigcup_{i=1}^{\infty} L_i$ is this regular?

$L =$ any non-regular language

$L \subseteq \Sigma^*$ countable

$L = \{w_1, w_2, w_3, \dots\}$

$L_i = \{w_i\}$

Fact Regular lang are NOT closed under infinite union.

$L_1 = \{$all strings containing "cs374"$\}$

$= \Sigma^* \{cs374\} \Sigma^*$

$L_2 = \{$all strings not containg cs374$\}$

$= \overline{L_1}$

$\emptyset \equiv$ empty language

$\varepsilon = \{\varepsilon\}$

$a = \{a\}$ $\forall a \in \Sigma$

r_1, r_2 regular expressions corresponding to reg lang R_1, R_2

$r_1 + r_2 \equiv R_1 \cup R_2$

$r_1 r_2 = R_1 R_2$

$r_1^* = R_1^*$ Kleene star

$r_1^3 = r_1 r_1 r_1$

$r_1^+ = r_1 r_1^*$

$(0+1)^* 111 (0+1)^*$ all binary strings containing 111 as a substring

$(0+1)^* 1 (0+1)^* 1 (0+1)^* 1 (0+1)^*$

$= \{$all bin strings with 111 as a subsequence$\}$

$(0+1) = 1+0$

$r^*s + t = ((r)^*s) + t$

$\emptyset = \{\}$

$\emptyset 0 = \{\}\{0\} = \{\}$

$= \emptyset$

$(\varepsilon+1)(01)^*(\varepsilon+0) \equiv$

$\boxed{01^* \neq (01)^*}$

$(\varepsilon+1)(01)^*(\varepsilon+0)$

ε

$(\varepsilon+0)(1+10)^*$

$(\varepsilon+0)(1+10)^* =$ all strings not containing 00 as a substring.

1 strings containing 101 or 010

$(0+1)^* 101 (0+1)^* + (0+1)^* 010 (0+1)^*$

All string with even # of 1s

$(\varepsilon + 11)^*$

$(0^* + 0^*$ * * * $^*10)^*$

$\Sigma^*(010 + 101)\Sigma^*$

$(0+1)^*(010+101)(0+1)^*$