
Prove that each of the following languages is **not** regular.

- 1 $\{0^{2^n} \mid n \geq 0\}$.
- 2 $\{0^{2n}1^n \mid n \geq 0\}$
- 3 $\{0^m1^n \mid m \neq 2n\}$
- 4 Strings over $\{0, 1\}$ where the number of 0s is exactly twice the number of 1s.
- 5 Strings of properly nested parentheses $()$, brackets $[]$, and braces $\{\}$. For example, the string $([]) \{\}$ is in this language, but the string $([])$ is not, because the left and right delimiters don't match.
- 6 Strings of the form $w_1\#w_2\#\dots\#w_n$ for some $n \geq 2$, where each substring w_i is a string in $\{0, 1\}^*$, and some pair of substrings w_i and w_j are equal.

Extra problems

- 7 $\{0^{n^2} \mid n \geq 0\}$
- 8 $\{w \in (0 + 1)^* \mid w \text{ is the binary representation of a perfect square}\}$