# HW 3

Due on **Wednesday, September 14, 2022 at 9am CST**

**CS/ECE 374A: Intro. Algorithms & Models of Computation, Fall 2022** Version: **1.0**

**Submission instructions as in previous homeworks.**

## 5 (100 PTS.) String flip.

Let $\Sigma = \{0, 1\}$, and let $L \subseteq \Sigma^*$ be a regular language. For a string $s = s_1 \ldots s_n \in \Sigma^*$, let $s^R = s_n s_{n-1} \ldots s_1$ be the *reverse* of $s$. Consider the following language

$$L_8 = \left\{ x y^R z \;\middle|\; x, y, z \in \Sigma^*, |y| \leq 8, \text{ and } \quad xyz \in L \right\}.$$

Thus, if $0101\underline{00001111}0101 \in L$, then $0101\underline{11110000}0101 \in L_8$ as is $010100\underline{100}1110101 \in L_8$. Prove that $L_8$ is a regular language.

To this end, you are given a DFA $M$ for $L$ – provide an NFA $N$ for $L_8$. Describe formally how you construct $N$ from $M$, and argue why your construction is correct. A formal proof that your construction works is not required.

Hints: (A) Your NFA should use its ability to guess things, and remember constant amount of information (how?). (B) To build up to the solution consider special cases, and solve them first, such as: (i) $x = \varepsilon$, (ii) $z = \varepsilon$, and (iii) $|y| = 2$.

## 6 (100 PTS.) Highly irregular.

For each of the following languages prove that they are not regular using fooling sets. Here $\Sigma = \{0, 1\}$.

**6.A.** (30 PTS.) For a string $w = w_1 w_2 \ldots w_k$, let $\text{odd}(w) = w_1 w_3 w_5 \cdots$ be the string formed by the odd characters of $w$. Consider the language $L_A = \{ w \in (0 + 1)^* \mid \text{odd}(w) \text{ is a palindrome} \}$.

**6.B.** (30 PTS.) $L_B = \{ w \in \Sigma^* \mid 10^n 10^n 1 \text{ is a substring of } w, \text{ where } n \text{ is an integer} \}$.

**6.C.** (40 PTS.) $L_C = \left\{ 0^i 1^j \;\middle|\; i + j = k^2, \text{ where } k \text{ is an integer} \right\}$.