HW 1

Due on **Wednesday, August 31, 2022 at 9am CST**

CS/ECE 374A: Intro. Algorithms & Models of Computation, Fall 2022 Version: **1.1**

- Guidelines for homework submissions are specified here:
  https://courses.engr.illinois.edu/cs374/fa2022/a/info/hw_policies.html
  Read them **carefully** before submitting your homework.

Some important course policies

- **You may use any source at your disposal** – paper, electronic, or human-but you *must* cite *every* source that you use, and you *must* write everything yourself in your own words. See the academic integrity policies on the course web site for more details.
- **Avoid the Three Deadly Sins!** Any homework or exam solution that breaks any of the following rules will be given an *automatic zero*, unless the solution is otherwise perfect. Yes, we really mean it. We are not trying to be scary or petty (Honest!), but we do want to break a few common bad habits that seriously impede mastery of the course material.
  - Always give complete solutions, not just examples.
  - Always declare all your variables, in English. In particular, always describe the specific problem your algorithm is supposed to solve.
  - Short **complete** answers are better than longer answers. Unnecessarily long answers (which by definition are not perfect) would get zero (i.e., 0) points. Avoid empty expressions like "in fact", "as anybody, or their uncle, can see if they think about it...", etc.
  - Always give credit to outside sources! (Yes, we are no good with counting.)

See the course web site for more information.

If you have any questions about these policies,
please do not hesitate to ask in class, in office hours, or on EdStem.

Extra problems (one fully solved) are available in the https://courses.engr.illinois.edu/cs374/fa2022/hw/hw_01_extra.pdfHW 1 extra problems collection on the class webpage: https://courses.engr.illinois.edu/cs374/fa2022. It is recommended that you look on these extra problems before doing the homework, since it would help you with doing the homeworks. These are also good practice problems for the midterms and final.

You also need to do (individually) also a question on PrairieLearn, which is due on **Tuesday, August 30, 2022 at 10am CST**.

1 (100 PTS.) Yo?

Let $\Sigma = \{0, 1\}$. A string $w \in \Sigma^*$ is ***yoyo*** if it does not contain the substring 00. Thus $w_1 =$ 01010101111 is yoyo, but 1010101001111 is not. Let Y be the language of all the yoyo strings.

1.A. (50 PTS.) Let $L \subseteq \Sigma^*$ be the language that is the minimal set with the following properties:

- $\varepsilon, 0, 1 \in L$.
- If $x, y \in L$, then $x1y \in L$.

Prove (by strong induction) that $L \subseteq Y$.

1.B. (50 PTS.) Prove (by strong induction) that $Y \subseteq L$ (thus $L = Y$).

2 (100 PTS.) Flipper in the bits.

Let $\Sigma = \{0, 1\}$, and consider the following pair of functions f, g defined over Σ^*:

$$f(w) = \begin{cases} \varepsilon & w = \varepsilon \\ \overline{c}g(x) & w = cx \end{cases} \qquad \text{and} \qquad g(w) = \begin{cases} \varepsilon & w = \varepsilon \\ cf(x) & w = cx, \end{cases}$$

where $c \in \Sigma$, and $\overline{c} = \begin{cases} 1 & c = 0 \\ 0 & c = 1. \end{cases}$ For example, we have

$$f(01100) = 1g(1100) = 11f(100) = 110g(00) = 1100f(0) = 11001g(\varepsilon) = 11001.$$

Similarly, we have $g(01100) = 0f(1100) = 00g(100) = 001f(00) = 0011g(0) = 00110f(\varepsilon) = 00110$.

2.A. (30 PTS.) Give a self contained recursive definition for f that does not involve g.

2.B. (70 PTS.) Prove the following identity for all strings w and x:

$$f(w \bullet x) = \begin{cases} f(w)f(x) & |w| \text{ is even} \\ f(w)g(x) & |w| \text{ is odd.} \end{cases}$$

You may assume without proof any result proved in class, in lab, or in the lecture notes. Otherwise, your proofs must be formal and self-contained, and they must invoke the formal definitions of concatenation $\bullet$, length $|\cdot|$, and the f and g functions. Do not appeal to intuition! In particular, you can use the following claim without proof:

Claim 1.1. *For any string $w = w_1 w_2 \cdots w_n \in \Sigma^*$, with $w_i \in \Sigma$, for all i, and for any index j, $1 \leq j < n$, we have $(w_1 \cdots w_j) \bullet (w_{j+1} \cdots w_n) = w$.*