LECTURE - 7

HW3 due at 8pm tonight

HW4 is out,
GPS is out

Last class

$$\begin{tikzcd} \text{DFA} \arrow[r, bend left, "\text{trivial}"] & \text{NFA} \arrow[l, bend left, "\text{subset construction}"] \end{tikzcd}$$

Today:

- $$\begin{tikzcd} \text{NFA} \arrow[r, bend right, "\text{(very high level idea)}"'] & \text{regular expressions} \arrow[l, bend right, "\text{Thompson's algorithm}"'] \end{tikzcd}$$

- Language Transformations

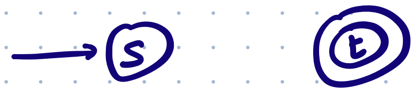
THOMPSON'S ALGORITHM

Regular expression $\longrightarrow$ ε-NFA $\left(\cdots \to \text{NFA} \to \text{DFA} \right)$

GOAL: Given a regular expression R
compute an NFA N
s.t. $L(R) = L(N)$

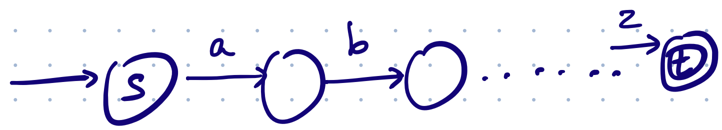
Regular Expressions

1) $R = \phi$



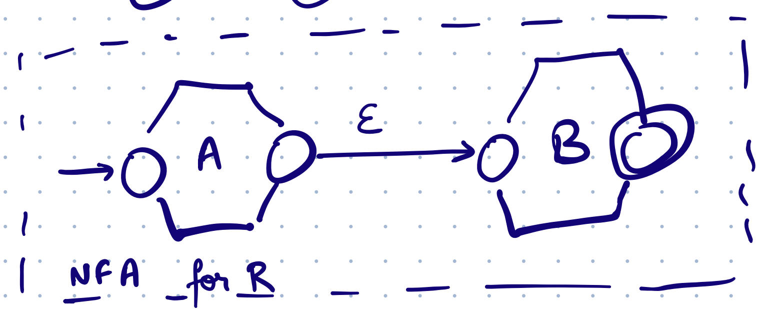
2) $R = w$ (some string)

$= abcd\dots z$

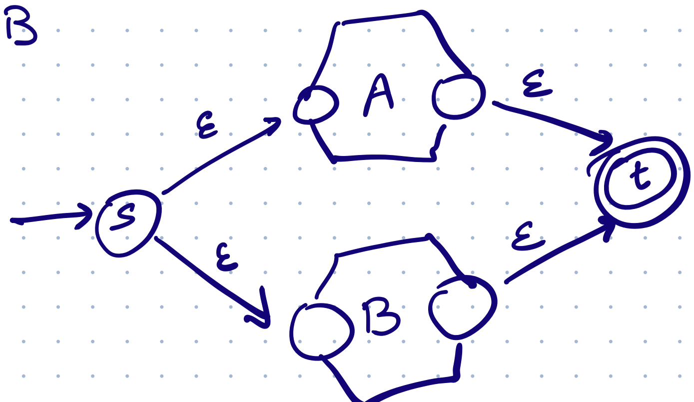


3) Concatenation

$R = A \cdot B$



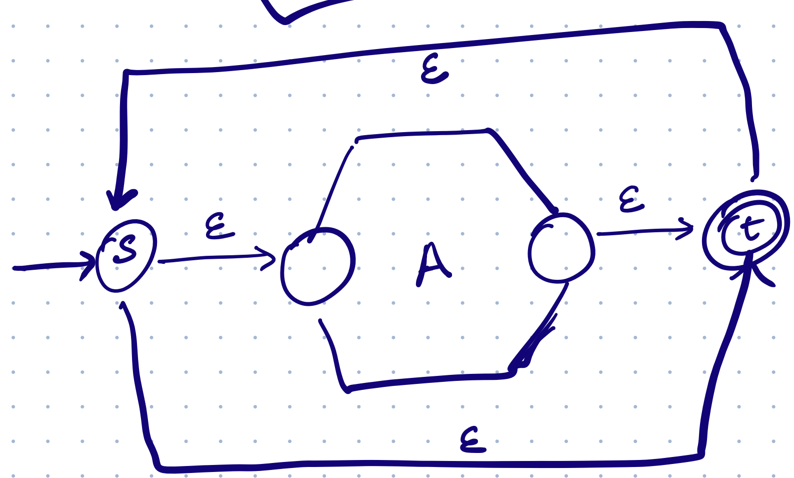
4) $R = A + B$



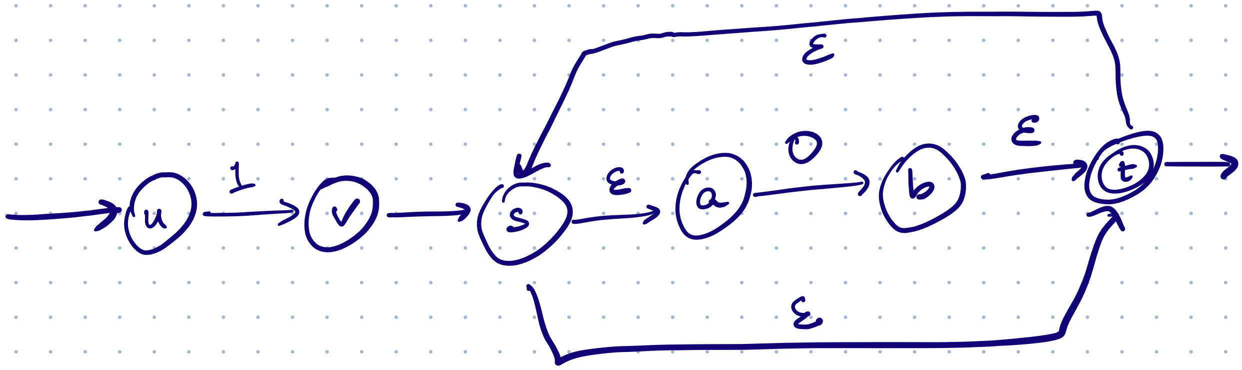
5) $R = A^*$

$= \varepsilon + A + A \cdot A$

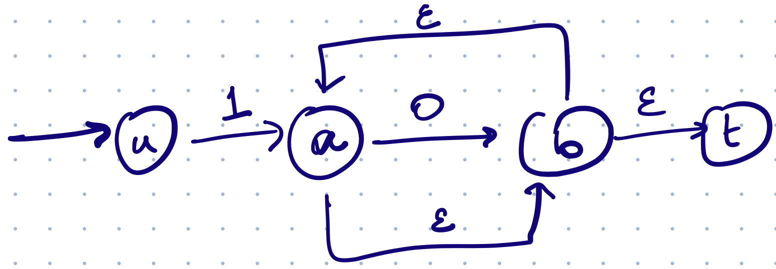
$+ \dots$



Example 10^*

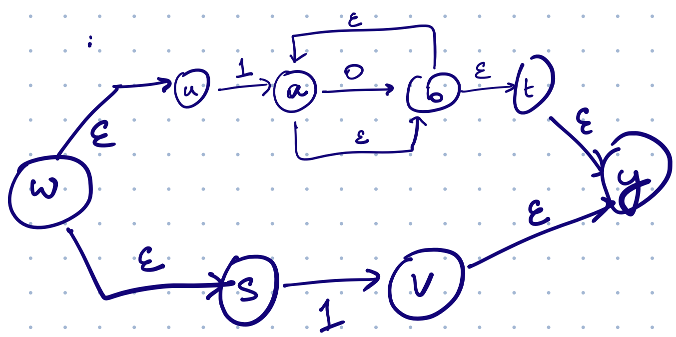


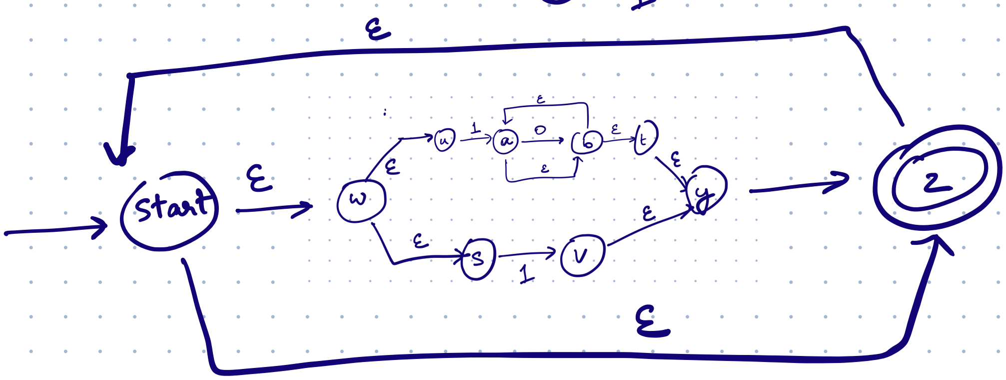
10^*:



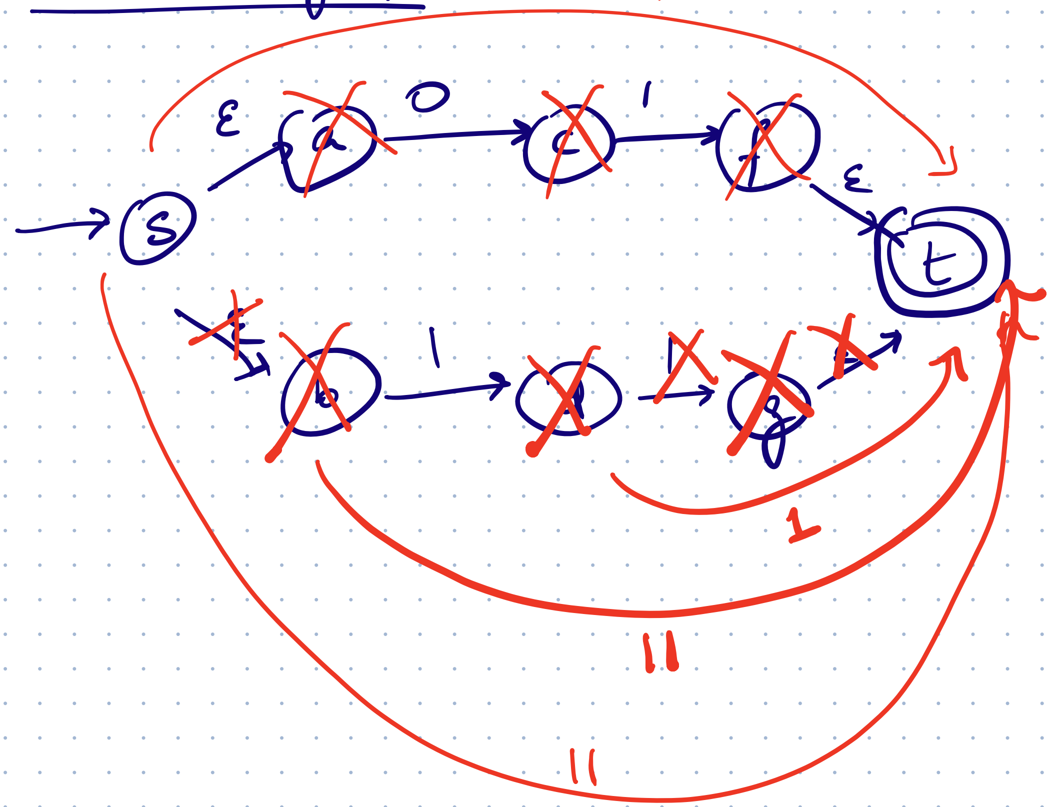
$(10^* + 1)^*$

$(10^* + 1)$:

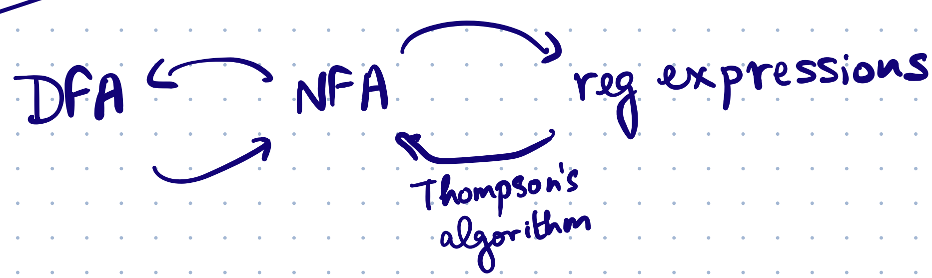




NFA → Regexp



Kleene's Theorem



Language Transformations

Given a regular language L,
prove that $T(L)$ is regular.

- $T(L) = \Sigma^* \setminus L$.

Given any regular L, there is a DFA for L.
Given this DFA $M = (Q, s, A, \delta)$
build DFA $\overline{M} = (\overline{Q}, \overline{s}, \overline{A}, \overline{\delta})$ for $T(L)$.

$\overline{Q} = Q$, $\overline{s} = s$, $\overline{A} = Q \setminus A$, $\overline{\delta} = \delta$

(Intuitively: swap Acc/reject states of M)

String reversal

- $\text{reverse}(L) = \{ w^R \mid w \in L \}$

Prove: If L is regular, so is $\text{reverse}(L)$

Given DFA $M = (Q, s, A, \delta)$ that accepts L.
We will build $M' = (Q^R, s^R, A^R, \delta^R)$
NFA that accepts $\text{reverse}(L)$.

INTUITION

Turn accepting states of DFA into start states of NFA

Start states to accept states

reverse arrows.

Given DFA $M = (Q, s, A, \delta)$

NFA $N' = (Q^R, s^R, A^R, \delta^R)$

$Q^R = Q \cup \{s^R\}$

$A^R = \{s\}$

s^R = new state ~~= A~~

$\delta^R(q, a) = \{p \mid \delta(p, a) = q\} \quad \forall q \in Q, a \in \Sigma$

$\delta^R(s^R, \varepsilon) = A$

$\delta^R(s^R, a) = \emptyset \quad \forall a \in \Sigma$

$\delta^R(q, \varepsilon) = \emptyset \quad \forall q \in Q$

$L = \{CATTAC, DOGGOD\}$ $T(L) = \{CAT, DOG\}$ $T(L) \cdot T(L)^R = \{CATGOD, CATTAC, \cdots\}$

Claim: $T(L) = \{w \mid ww^R \in L\}$

If L is regular, prove that $T(L)$ is also regular.

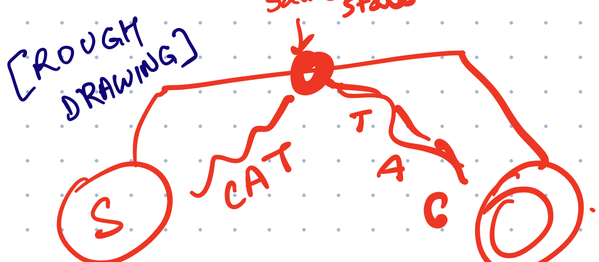
$$\boxed{\text{Intuition} \quad CAT \in T(L) \iff CATTAC \in L.}$$

Given DFA for L, $M = (Q, s, A, \delta)$

Build DFA/NFA for $T(L)$, $M' = (Q', s', A', \delta')$.

Run both M, M^R in parallel.

same state ← meet in the middle



M' is a product construction of DFA M and NFA M'.

$$Q' = Q \times (Q \cup \{s^R\})$$

(p, q)
$p \in Q$,
$q \in Q \cup \{s^R\}$.

$$s' = (s, s^R)$$

$$A' = \{(q, q) \mid q \in Q\}$$

$$\boxed{\delta'((s, s^R), \varepsilon) = \{(s, q) \mid q \in A\}}$$

M^R

$\delta'((q,r),\varepsilon) = \phi \quad \forall (q,r) \in Q' \quad (q,r) \neq (s,s^R)$

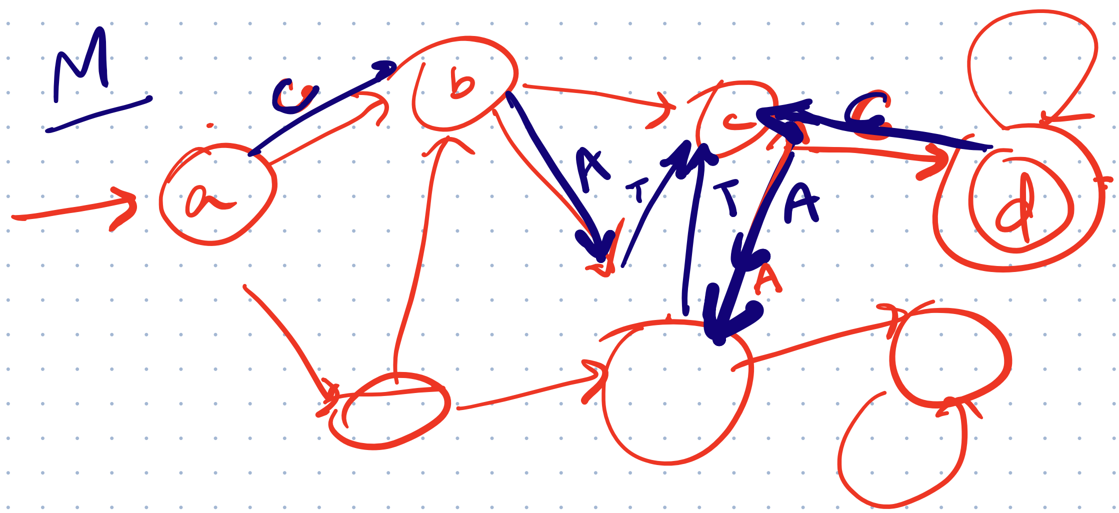
transition function of M^R

$\delta'((q,r),a) = \{(\delta(q,a), p) \mid \delta(p,a) = r\}$

$\delta'((q,s^R),a) = \phi. \quad \forall q \in Q$

[ROUGH DRAWINGS]

M



M'