LECTURE - 3

HW1 due 8 pm tonight

OH on course webpage

FINITE STATE MACHINES

DETERMINISTIC FINITE AUTOMATA

non-empty ← Q - finite set of states

$s \in Q$ - Start state

$A \subseteq Q$ - Accepting states

Σ - input alphabet — finite set, eg $\Sigma = \{0,1\}$

$\delta : Q \times \Sigma \to Q$ - transition function

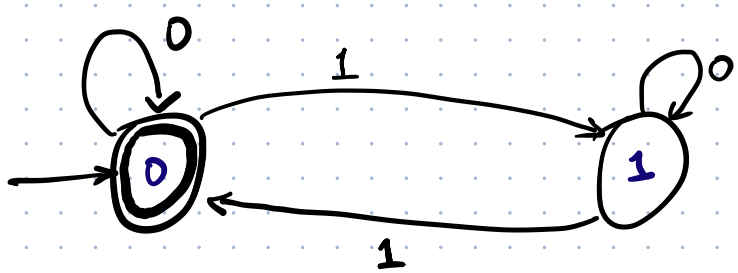
$L = \{$ strings that contain an even number of 1s $\}$

```
EVENONES (w[1....n])
  ones ← 0
    for i = 1 to n
      if w[i]=1, ones ← (1+ ones) mod 2
  if ones =
  else   FALSE
```

state → ones is either 0 or 1

Accept →

01011



$$\delta[q, a] = (q + a) \bmod 2$$

	q	$\delta[q, 0]$	$\delta[q, 1]$	$Accept[q]$
START STATE →	0	0	1	✓ TRUE
	1	1	0	FALSE

DoSomethingCool$(\delta[\ ,\ ], A[\ ], w[1 \ldots n])$

$q \leftarrow 0$
for $i = 1$ to n
  $q \leftarrow \delta[q, w[i]]$
return $A[q]$.

EXTENDED STATE TRANSITION FUNCTION

$$\delta^* : Q \times \Sigma^* \longrightarrow Q$$

$$\delta^*(q, w) = \begin{cases} q & w = \varepsilon \\ \delta^*(\delta(q, a), x) & w = a.x \end{cases}$$

$$= \begin{cases} q & w = \varepsilon \\ \delta(\delta^*(q, x), a) & w = x.a \end{cases}$$

M accepts $w \iff \delta^*(\text{start}, w) \in A$

$M = Q, s \in Q, A \subseteq Q, \Sigma, \delta$ (or δ^*)

$L(M) = \{ w \in \Sigma^* \mid M \text{ ACCEPTS } w \}$.

Is this binary number (w) divisible by 5?

```
MULTIPLEOF5(w[1..n]):
  rem ← 0
  for i ← 1 to n
      rem ← (2 · rem + w[i]) mod 5
  if rem = 0
      return TRUE
  else
      return FALSE
```

state (5 possibilities)

transition function

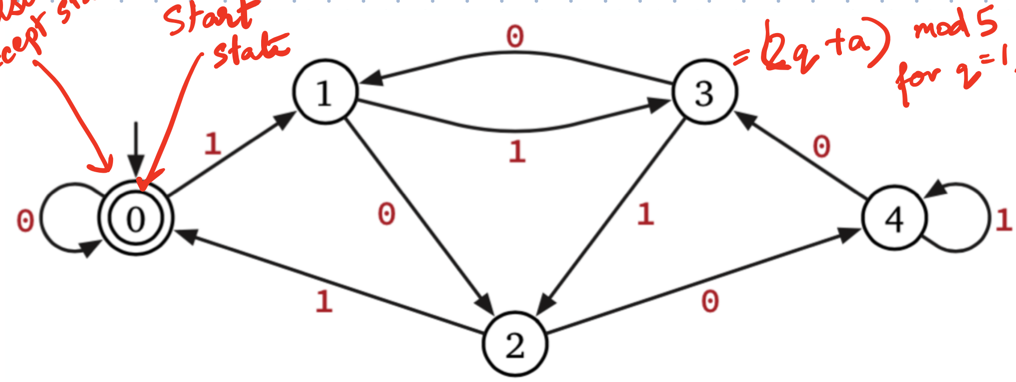
accepting condition

$$\boxed{\delta(q, a) = (2q + a) \bmod 5.}$$

also accept state

Start state

$= (2q + a) \bmod 5$ for $q = 1, a = 1$.



State-transition graph for MULTIPLEOF5

q	$\delta[q, 0]$	$\delta[q, 1]$	$A[q]$
0	0	1	TRUE
1	2	3	FALSE
2	4	0	FALSE
3	1	2	FALSE
4	3	4	FALSE

(found, last 2)

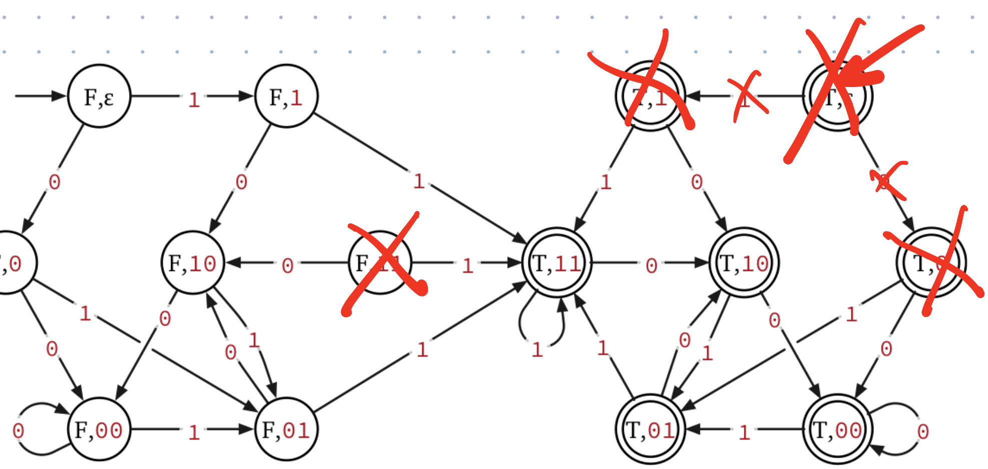
2 (T/F)   7

$\boxed{\text{14 states}}$

```
CONTAINS11(w[1..n]):
  found ← FALSE
  for i ← 1 to n
      if i = 1
          last2 ← w[1]
      else
          last2 ← w[i−1] · w[i]
      if last2 = 11
          found ← TRUE
  return found
```

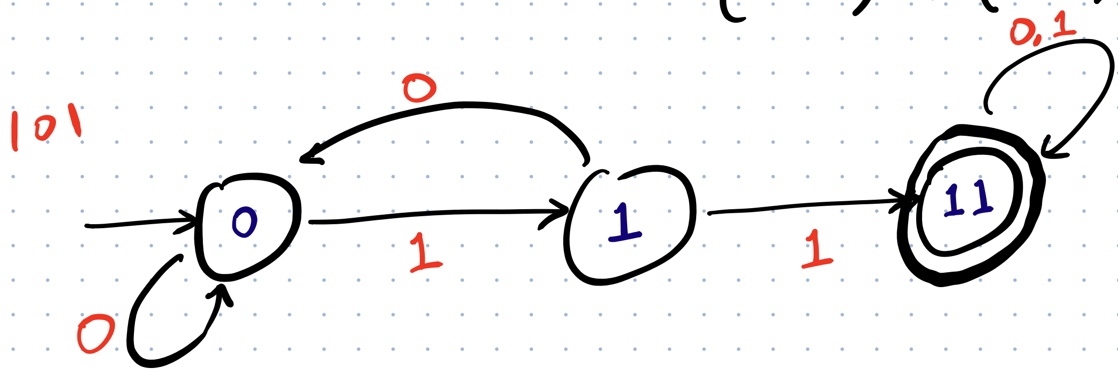
q	$\delta[q, 0]$	$\delta[q, 1]$
(FALSE, ε)	(FALSE, 0)	(FALSE, 1)
(FALSE, 0)	(FALSE, 00)	(FALSE, 01)
(FALSE, 1)	(FALSE, 10)	(**TRUE**, 11)
(FALSE, 00)	(FALSE, 00)	(FALSE, 01)
(FALSE, 01)	(FALSE, 10)	(**TRUE**, 11)
(FALSE, 10)	(FALSE, 00)	(FALSE, 01)
(FALSE, 11)	(FALSE, 10)	(**TRUE**, 11)

q	$\delta[q, 0]$	$\delta[q, 1]$
(TRUE, ε)	(TRUE, 0)	(TRUE, 1)
(TRUE, 0)	(TRUE, 00)	(TRUE, 01)
(TRUE, 1)	(TRUE, 10)	(TRUE, 11)
(TRUE, 00)	(TRUE, 00)	(TRUE, 01)
(TRUE, 01)	(TRUE, 10)	(TRUE, 11)
(TRUE, 10)	(TRUE, 00)	(TRUE, 01)
(TRUE, 11)	(TRUE, 10)	(TRUE, 11)



Our brute-force DFA for strings containing the substring 11

$L = \{ \text{strings containing } 11 \}$
$= (0+1)^* 11 (0+1)^*$



0 state : Last symbol (if any) is a 0,
have not seen 11.

1 state : Last symbol is a 1,
have not seen 11.

11 state : Have seen 11.

EXAMPLE OF A LANGUAGE THAT IS NOT REGULAR?

$L = \{ 0^n 1^n : n \text{ is an integer} \geq 0 \}$.