HW 11 will be out later today (due 2 weeks from now)

GPS 11 will be out later today.

P - solvable in polynomial time

NP - checkable in polynomial time.

X is NP-hard - If poly time algo for X, then P = NP

∴ existence of this is also really unlikely

really unlikely.

To prove X is NP-hard:

Pick a known NP-hard problem

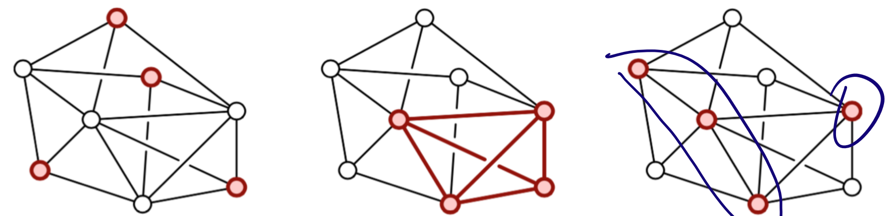
(Cook-Levin : Circuit-SAT is NP-hard)

CSAT $\leq_p$ 3SAT ∴ 3SAT is NP-hard

3SAT $\leq_p$ MaxClique ∴ MaxClique is NP-hard

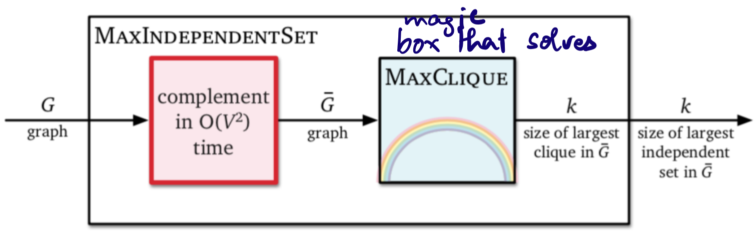
3SAT $\leq_p$ MAXINDSET ∴ Max IndepSet is NP-hard

MaxINDSET $\leq_p$ MINVC : MinVerCov is NP-hard

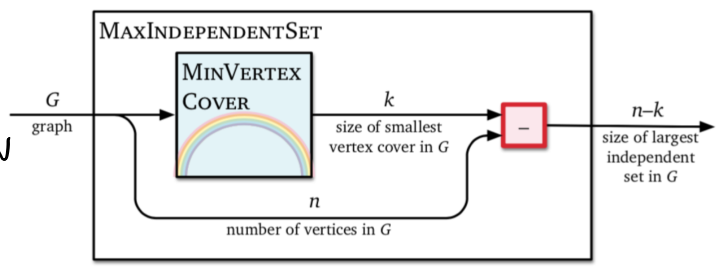


MAX. INDEPENDENT SET MAX CLIQU MIN VERTEX COVER

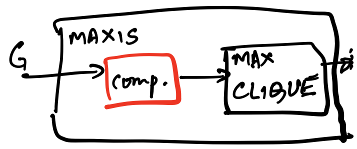
MAXINDSET $\leq_p$ MAXCLIQUE



MAXINDSET $\leq_p$ MINVERTEXCOV



(also MAXCLIQUE $\leq_p$ MAXINDSET)

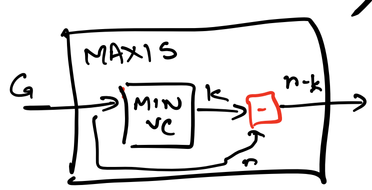


$X \leq_p Y$

$\not\Rightarrow$

$Y \leq_p X$

(also MIN VC $\leq_p$ MAXINDSET)



2SAT   3SAT

$2SAT \le_p 3SAT$

$(a \vee b) \rightarrow (a \vee b \vee c) \wedge (a \vee b \vee \bar{c})$

2SAT is easy.

3SAT is NP-hard.

$3SAT \le_p 2SAT$? Likely not true

(If true, then P=NP)

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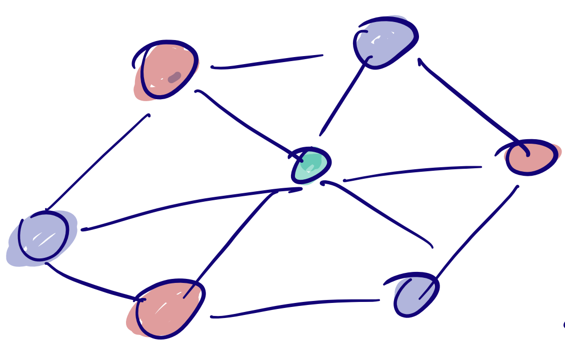
3 COLOR



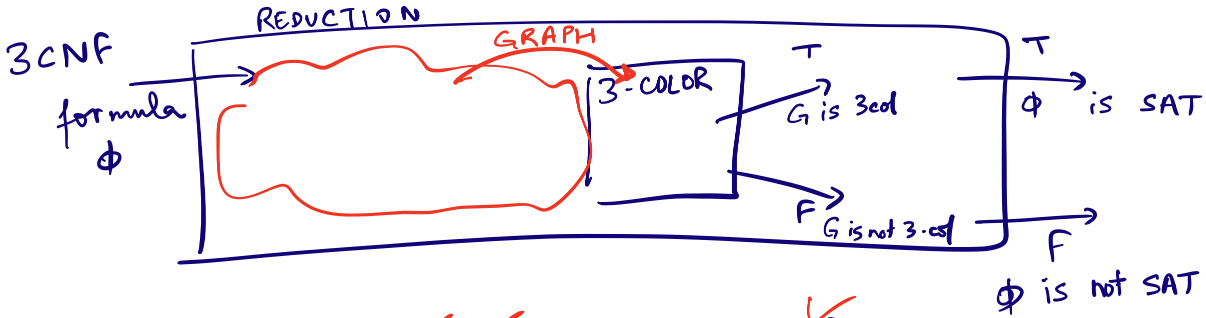
Input : $G=(V,E)$

Problem : Can we color vertices red, green, blue such that every edge touches two colors?

(no 2 vertices connected by an edge share the same color).

Claim: 3-COLOR is NP-hard.

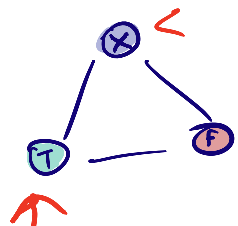
Proof: $3SAT \le_p 3\text{-COLOR}$



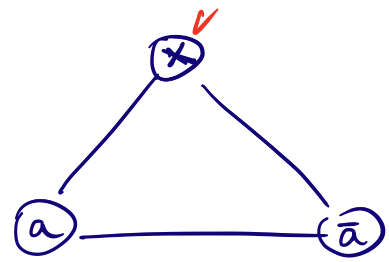
Formula : $(a \vee b \vee c) \wedge (b \vee \bar{c} \vee \bar{d}) \ldots$

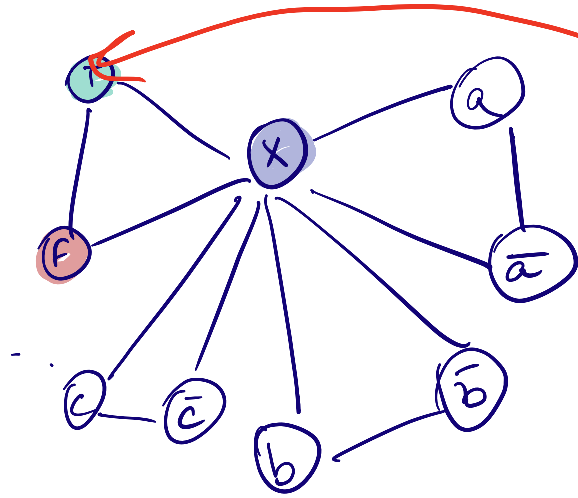
Graph :

Truth gadget

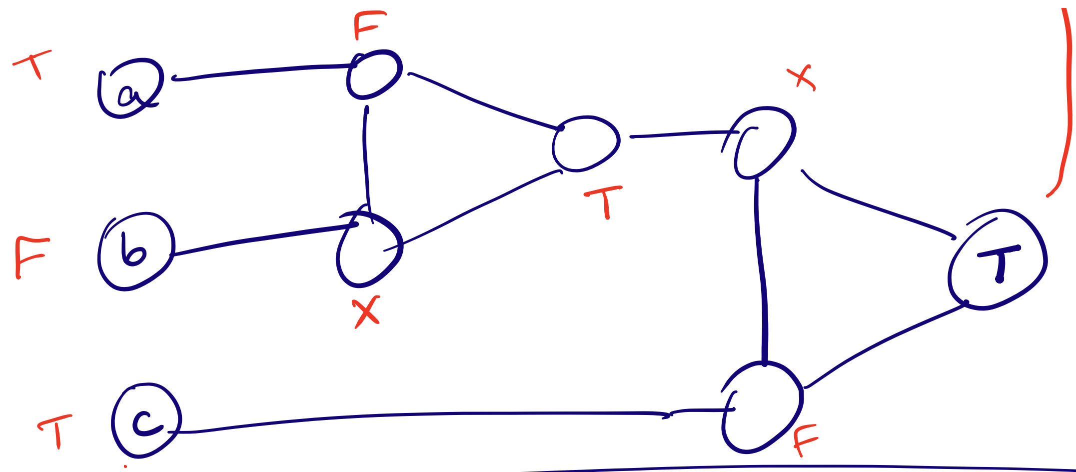


Variable gadget
(one for each variable)





Clause gadget : $(a \vee b \vee c)$



Claim 1: Clause gadget has a 3-coloring iff $\Longleftrightarrow$ there is at least 1 true literal in the clause

Proof by picture that if a, b, c are all false $\Rightarrow$ no 3COL exists

(by negation) If 3COL exists $\Rightarrow$ 1 of a, b, c must be true

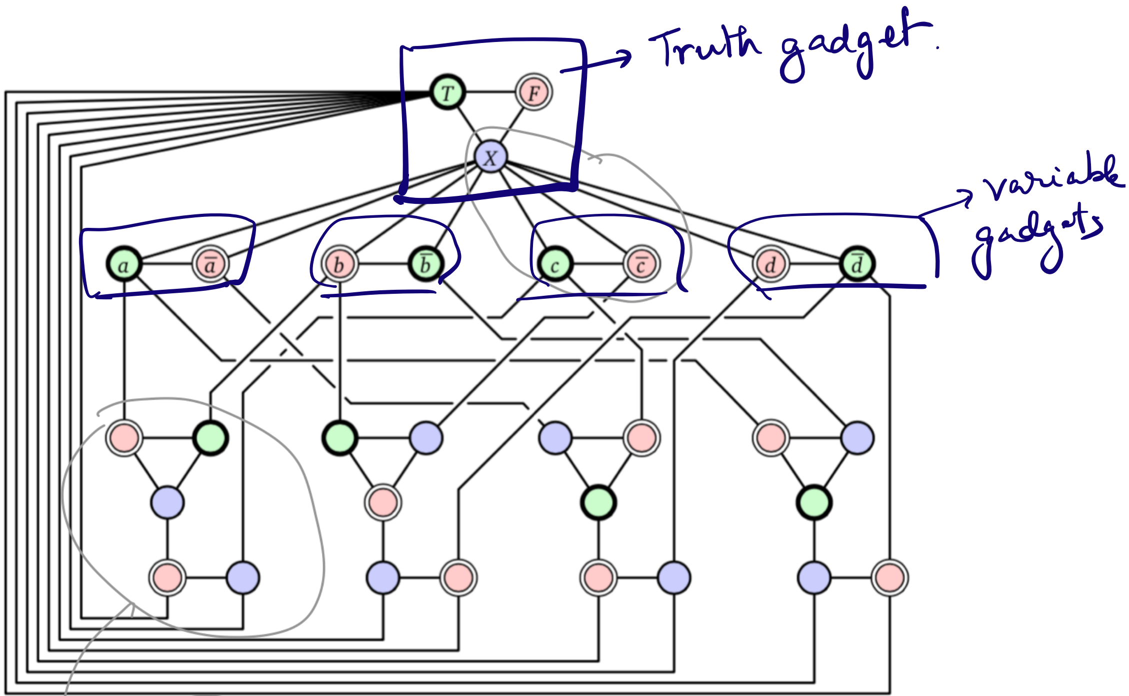
If 1 of a, b, c is true $\Rightarrow$ 3 COL exists.

Claim 1 $\Rightarrow$

Every clause must have at least 1 true literal if and only if G has a 3-COL

POLYNOMIAL - TIME. $\therefore$ 3SAT $\leq_p$ 3-COL

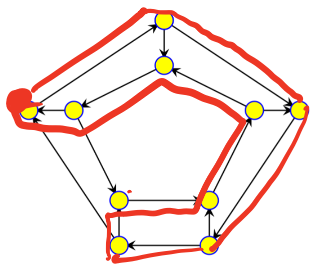
3-COL $\leq_p$ 4-COL $\leq_p$ 5-COL



$(a \vee b \vee c) \wedge (b \vee \bar{c} \vee \bar{d}) \wedge (\bar{a} \vee c \vee d) \wedge (a \vee \bar{b} \vee \bar{d})$

clause

HAMILTONIAN CYCLE



Given graph $G = (V, E)$

Is there a cycle that touches every vertex exactly once?

Claim: Hamiltonian cycle is NP-complete.

Proof: Hamiltonian cycle is in NP. (exercise)

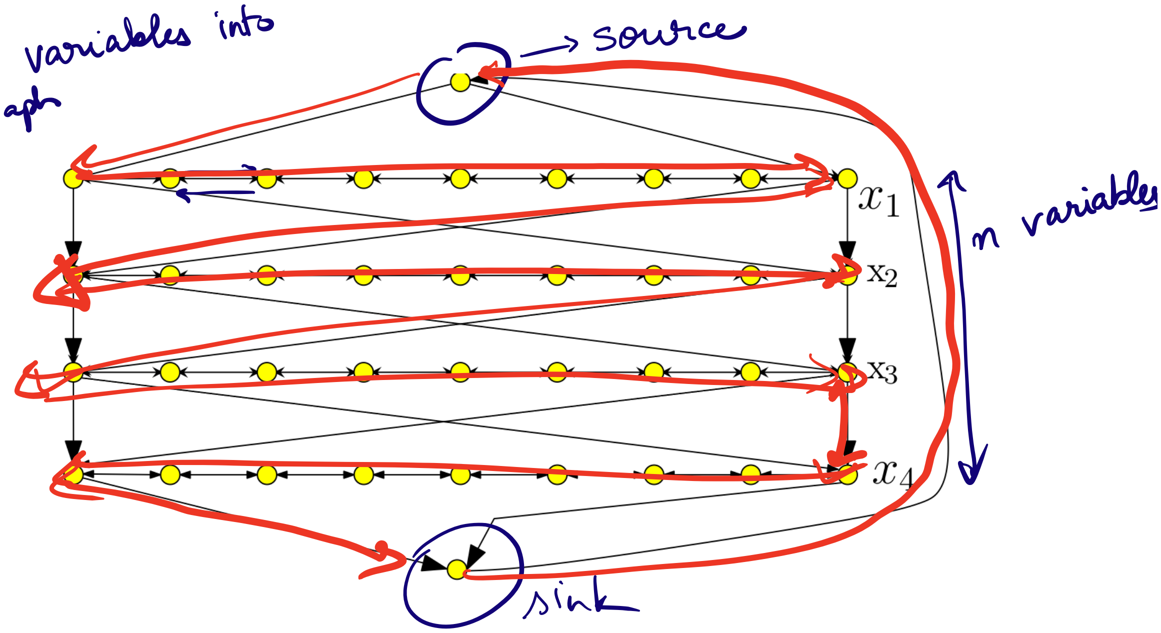
Ham. cycle is NP-hard.

$3SAT \le_p$ Ham cycle in directed graphs

Given 3-CNF formula $\varphi \Rightarrow$ convert to G.

s.t. G has Ham. cycle $\Leftrightarrow \varphi$ is satisfiable

Encoded variables into the graph

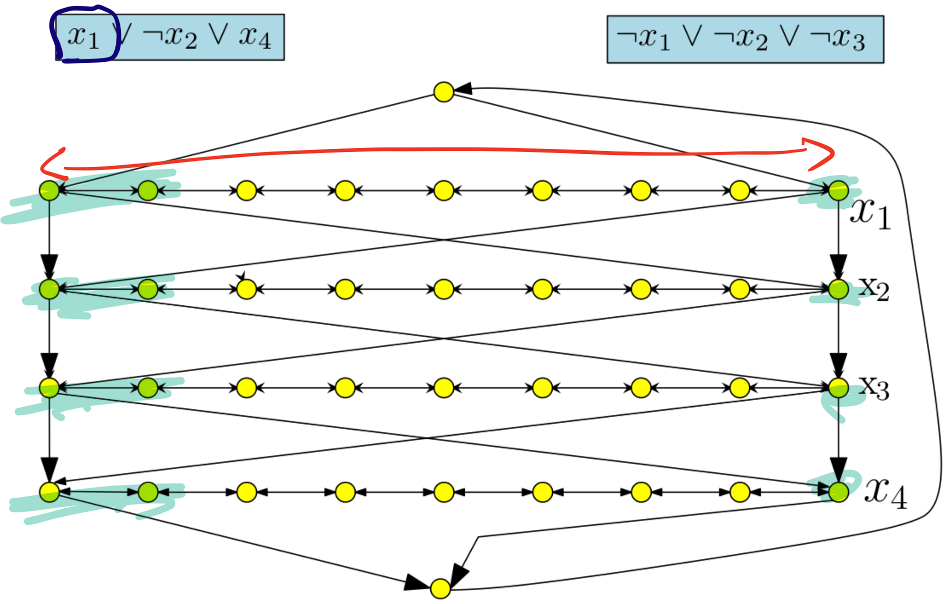


3CNF: formula: $(x_1 \vee \neg x_2 \vee x_4) \wedge (\neg x_1 \vee \neg x_2 \vee \neg x_3)$

2^n Hamiltonian cycles

Encode clause into the graph

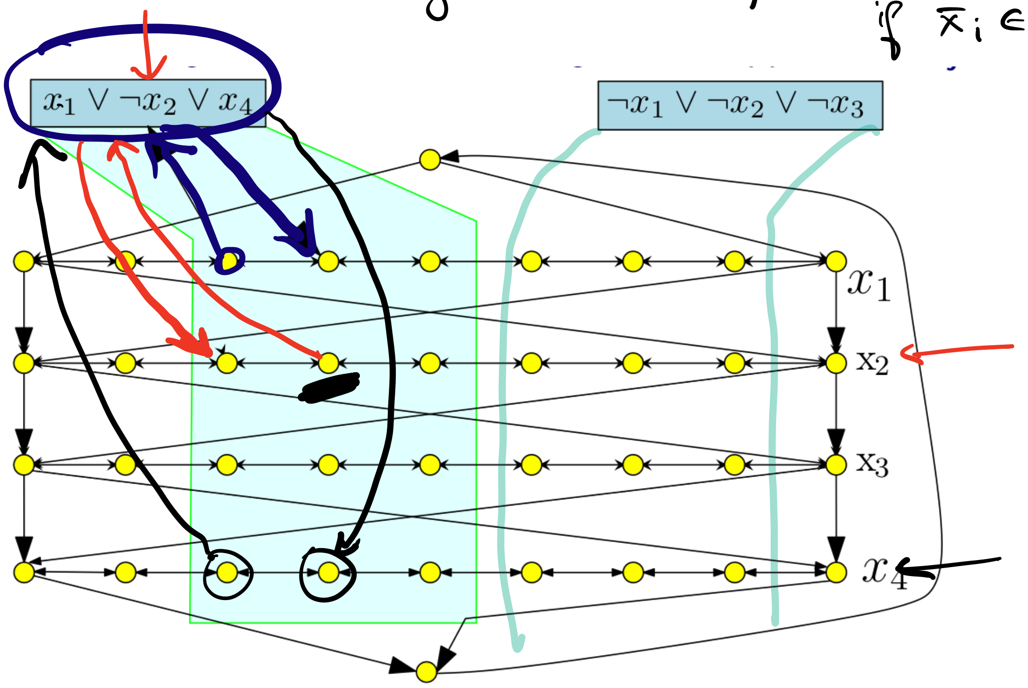
3k+3 vertices in each row, where k is number of clauses

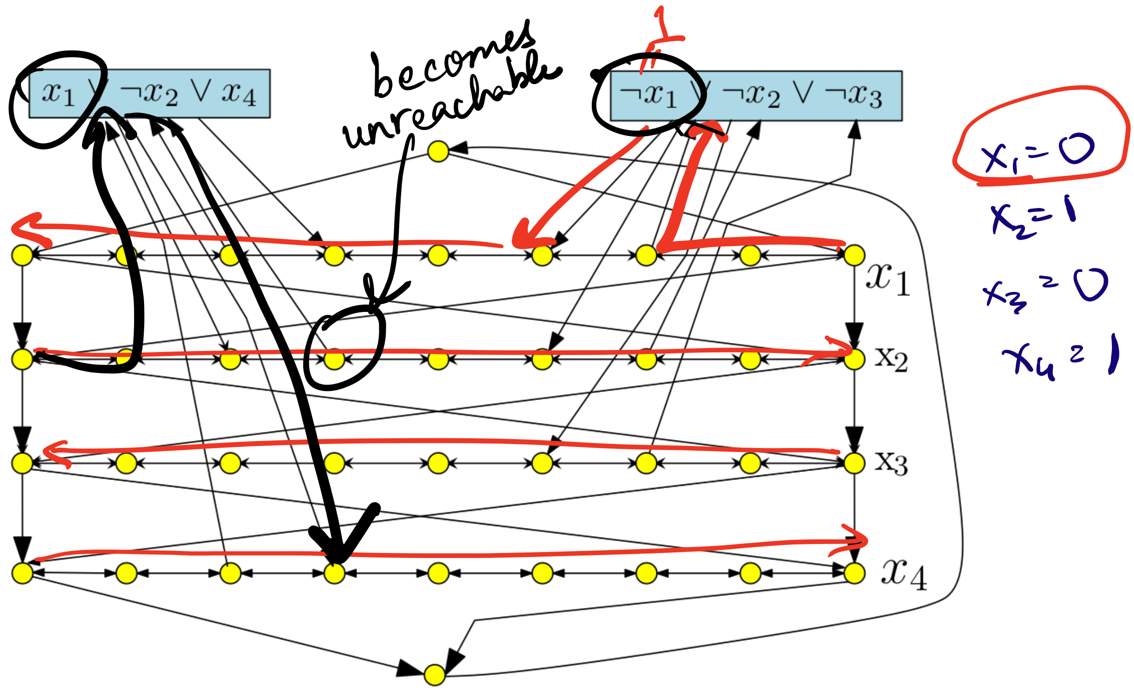


Add vertex c_j for every clause C_j.

c_j has edge from $3j$ and to $3j+1$ on i^{th} row if $x_i \in C_j$

c_j has " $3j+1$ and to $3j$ on i^{th} row if $\bar{x}_i \in C_j$.





Claim : There is a SAT assignment to the 3CNF formula $\Longleftrightarrow$ there is a Hamiltonian cycle on the graph.

proof.

$\Longrightarrow$ Suppo there is a SAT assignment.

$\Rightarrow$ there is a Ham. cycle.

Suppose there is a Ham. cycle

$\Rightarrow$ SAT assignment.

$\boxed{\text{Sub-claim}}$ Ham. cycle can make at most 1-hop detours on G.

(no funny business)

$\therefore$ traverse each row in a single direction