

For each statement below, check “Yes” if the statement is **ALWAYS** true and “No” otherwise, and give a **brief** explanation of your answer.

(a) Every integer in the empty set is prime.

Yes No

(b) The language $\{0^m 1^n \mid m + n \leq 374\}$ is regular.

Yes No

(c) The language $\{0^m 1^n \mid m - n \leq 374\}$ is regular.

Yes No

(d) For all languages L , the language L^* is regular.

Yes No

(e) For all languages L , the language L^* is infinite.

Yes No

(f) For all languages $L \subset \Sigma^*$, if L can be represented by a regular expression, then $\Sigma^* \setminus L$ is recognized by a DFA.

Yes No

(g) For all languages L and L' , if $L \cap L' = \emptyset$ and L' is not regular, then L is regular.

Yes No

(h) Every regular language is recognized by a DFA with exactly one accepting state.

Yes No

(i) Every regular language is recognized by an NFA with exactly one accepting state.

Yes No

(j) Every language is either regular or context-free.

Yes No

For each of the following languages over the alphabet $\Sigma = \{0, 1\}$, either *prove* that the language is regular or *prove* that the language is not regular. **Exactly one of these two languages is regular.** Both of these languages contain the string 00110100000110100.

1. $\{0^n w 0^n \mid w \in \Sigma^+ \text{ and } n > 0\}$

2. $\{w 0^n w \mid w \in \Sigma^+ \text{ and } n > 0\}$

The *parity* of a bit-string w is 0 if w has an even number of 1 s, and 1 if w has an odd number of 1 s. For example:

$$\text{parity}(\varepsilon) = 0 \quad \text{parity}(0010100) = 0 \quad \text{parity}(00101110100) = 1$$

- (a) Give a *self-contained*, formal, recursive definition of the *parity* function. (In particular, do **not** refer to $\#$ or other functions defined in class.)
- (b) Let L be an arbitrary regular language. Prove that the language $\text{OddParity}(L) := \{w \in L \mid \text{parity}(w) = 1\}$ is also regular.
- (c) Let L be an arbitrary regular language. Prove that the language $\text{AddParity}(L) := \{\text{parity}(w) \cdot w \mid w \in L\}$ is also regular.

[Hint: Yes, you have enough room.]

CS/ECE 374 A ✧ Fall 2021
Fake Midterm 1 Problem 4

Name:

For each of the following languages L , give a regular expression that represents L **and** describe a DFA that recognizes L . You do **not** need to prove that your answers are correct.

(a) All strings in $(0 + 1)^*$ that do not contain the substring 0110 .

(b) All strings in 0^*10^* whose length is a multiple of 3.

For any string $w \in \{0, 1\}^*$, let $oblivate(w)$ denote the string obtained from w by removing every 1. For example:

$$\begin{aligned}oblivate(\varepsilon) &= \varepsilon \\oblivate(000000) &= 000000 \\oblivate(111111) &= \varepsilon \\oblivate(010001101) &= 00000\end{aligned}$$

Let L be an arbitrary regular language.

1. **Prove** that the language $OBLIVATE(L) = \{oblivate(w) \mid w \in L\}$ is regular.

2. **Prove** that the language $UNOBLIVATE(L) = \{w \in \{0, 1\}^* \mid oblivate(w) \in L\}$ is regular.