CS 374 A

08/24/21

```
COLLATZ(n):
  if n = 1
      return TRUE
  if n is even
      n ← n/2
  else  n ← 3n+1
```

INDUCTION / RECURSION

STRINGS sequence of characters/symbols of an alphabet.

Σ is a finite set. $\Sigma = \{0,1\}$

A string is
- either empty ε
- or $a \cdot x$ for $a \in \Sigma$ and string x

STRING = S.(TRING) = S.(T.(RING))

Length $|w|$ of string w is

$$|w| = \begin{cases} 0 & \text{if } w = \varepsilon \\ 1 + |x| & \text{if } w = a \cdot x \end{cases}$$

Concatenation . alternatively

$w \| z$ $\quad (w \bullet z) \to$ BIG DOT

$w =$ CAT $\quad z =$ FISH

$w \| z = \underline{\text{CAT}} \| \underline{\text{FISH}} = \text{C} \cdot \text{AT} \| \text{F} \cdot \text{ISH}$
$= \text{C} \cdot (\text{AT} \| \text{FISH})$

$$w \| z = \begin{cases} z & \text{if } w = \varepsilon \\ a \cdot (x \| z) & \text{if } w = a \cdot x \end{cases}$$

$$\begin{aligned} \text{CAT} \| \text{FISH} &= \text{C} \cdot (\text{AT} \| \text{FISH}) \\ &= \text{C} \cdot (\text{A} \cdot (\text{T} \| \text{FISH})) \\ &= \text{C} \cdot (\text{A} \cdot (\text{T} \cdot (\varepsilon \| \text{FISH}))) \\ &= \text{C} \cdot (\text{A} \cdot (\text{T} \cdot (\text{FISH}))) \\ &= \text{C} \cdot (\text{A} \cdot \text{TFISH}) \\ &= \text{C} \cdot (\text{ATFISH}) = \text{CATFISH} \end{aligned}$$

Theorem: For all strings w and z,

$$|w\|z| = |w| + |z|.$$

Proof:

Let w and z be 2 arbitrary strings.

Recall: $w\|z = \begin{cases} z & \text{if } w = \varepsilon \\ a\cdot(x\|z) & \text{if } w = a\cdot x \end{cases}$

I.H.: Assume $|x\|z| = |x| + |z|$, for all strings x s.t. $|x| < |w|$, for all strings z.

There are 2 cases

$w = \varepsilon$. $|w\|z| = |\varepsilon\|z|$ ($\because w = \varepsilon$)

$= |z|$ (def. $\|$)

$= 0 + |z| = |\varepsilon| + |z|$ (def 1.1 LENGTH)

$= |w| + |z|$ (Because $w = \varepsilon$)

$w = a\cdot x$ $|w\|z| = |(a\cdot x)\|z|$ ($\because w = a\cdot x$)

$= |a\cdot(x\|z)|$ (def $\|$)

$= 1 + |(x\|z)|$ (def LENGTH)

$= 1 + |x| + |z|$ (by I.H.)

$$= |a \cdot x| + |z| \quad \text{(by LENGTH)}$$

$$= |w| + |z|. \quad (\text{by } w = a \cdot x)$$

So, $|w||z| = |w| + |z|$

BOILERPLATE.

Thm: Every string is zowzy.

Proof. Let w be an arbitrary string.

I.H. Assume for every x where $|x| < |w|$, that x is zowzy.

2 cases:

If $w = \varepsilon$

w is zowzy

If $w = a \cdot x$

w is zowzy

In Both cases, w is zowzy.