

CS 374 A

08/24/21

COLLATZ(n):

```
if  $n=1$ 
  return TRUE
if  $n$  is even
   $n \leftarrow n/2$ 
else
   $n \leftarrow 3n+1$ 
```

INDUCTION / RECURSION

STRINGS sequence of characters/
symbols
of an alphabet.

Σ is a finite set. $\Sigma = \{0,1\}$

A string is

- either empty ϵ

- or $a \cdot x$ for $a \in \Sigma$ and string x

STRING = S.(TRING) = S.(T.(RING))

Length $|w|$ of string w is

$$|w| = \begin{cases} 0 & \text{if } w = \epsilon \\ 1 + |x| & \text{if } w = a \cdot x \end{cases}$$

Concatenation. alternatively

$$w \parallel z \quad (w \cdot z) \rightarrow \text{BIG DOT}$$

$$w = \text{CAT} \quad z = \text{FISH}$$

$$w \parallel z = \underline{\text{CAT}} \parallel \underline{\text{FISH}} = \text{C} \cdot \text{AT} \parallel \text{F} \cdot \text{ISH} \\ = \text{C} \cdot (\text{AT} \parallel \text{FISH})$$

$$w \parallel z = \begin{cases} z & \text{if } w = \epsilon \\ a \cdot (x \parallel z) & \text{if } w = a \cdot x \end{cases}$$

$$\begin{aligned} \text{CAT} \parallel \text{FISH} &= \text{C} \cdot (\text{AT} \parallel \text{FISH}) \\ &= \text{C} \cdot (\text{A} \cdot (\text{T} \parallel \text{FISH})) \\ &= \text{C} \cdot (\text{A} \cdot (\text{T} \cdot (\epsilon \parallel \text{FISH}))) \\ &= \text{C} \cdot (\text{A} \cdot (\text{T} \cdot (\text{FISH}))) \\ &= \text{C} \cdot (\text{A} \cdot \text{TFISH}) \\ &= \text{C} \cdot (\text{ATFISH}) = \text{CATFISH} \end{aligned}$$

Theorem: For all strings w and z ,

$$|w||z| = |w| + |z|.$$

Proof:

Let w and z be 2 arbitrary strings.

Recall: $w||z = \begin{cases} z & \text{if } w = \epsilon \\ a \cdot (x||z) & \text{if } w = a \cdot x \end{cases}$

I.H.: Assume $|x||z| = |x| + |z|$, For all strings x st. $|x| < |w|$, For all strings z .

There are 2 cases

$w = \epsilon$. $|w||z| = |\epsilon||z|$ ($\because w = \epsilon$)

$= |z|$ (def. ||)

$= 0 + |z| = |\epsilon| + |z|$

$= |w| + |z|$ (def. |·| LENGTH)

(Because $w = \epsilon$)

$w = a \cdot x$

$|w||z| = |a \cdot x||z|$ ($\because w = a \cdot x$)

$= |a \cdot (x||z)|$ (def. ||)

$= 1 + |(x||z)|$ (def. LENGTH)

$= 1 + |x| + |z|$ (by I.H.)

$$= |a \cdot x| + |z| \quad (\text{by LENGTH})$$

$$= |w| + |z|. \quad (\text{by } w = a \cdot x)$$

$$\text{So, } |w| + |z| = |w| + |z|$$

BOILERPLATE.

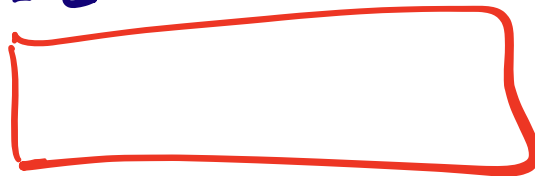
Thm: Every string is **zowzy**.

Proof. Let w be an arbitrary string.

I.H. Assume for every x where $|x| < |w|$,
that x is **zowzy**.

2 cases:

If $w = \epsilon$



w is zowzy

If $w = a \cdot x$



w is zowzy

In Both cases, w is **zowzy**.