

You saw the following context-free grammars in class on Thursday; in each example, the grammar itself is on the left; the explanation for each non-terminal is on the right.

- Properly nested strings of parentheses.

$$S \rightarrow \varepsilon \mid S(S) \quad \text{properly nested parentheses}$$

Here is a different grammar for the same language:

$$S \rightarrow \varepsilon \mid (S) \mid SS \quad \text{properly nested parentheses}$$

- $\{0^m 1^n \mid m \neq n\}$. This is the set of all binary strings composed of some number of 0s followed by a *different* number of 1s.

$$\begin{array}{ll} S \rightarrow A \mid B & \{0^m 1^n \mid m \neq n\} \\ A \rightarrow 0A \mid 0C & \{0^m 1^n \mid m > n\} \\ B \rightarrow B1 \mid C1 & \{0^m 1^n \mid m < n\} \\ C \rightarrow \varepsilon \mid 0C1 & \{0^m 1^n \mid m = n\} \end{array}$$

Give context-free grammars for each of the following languages over the alphabet $\Sigma = \{0, 1\}$. For each grammar, describe the language for each non-terminal, either in English or using mathematical notation, as in the examples above. We probably won't finish all of these during the lab session.

1. All palindromes in Σ^*
2. All palindromes in Σ^* that contain an even number of 1s
3. All palindromes in Σ^* that end with 1
4. All palindromes in Σ^* whose length is divisible by 3
5. All palindromes in Σ^* that do not contain the substring 00

Harder problems to work on later:

6. $\{0^{2n}1^n \mid n \geq 0\}$

7. $\{0^m1^n \mid m \neq 2n\}$

[Hint: If $m \neq 2n$, then either $m < 2n$ or $m > 2n$. Extend the previous grammar, but pay attention to parity. This language contains the string 01.]

8. $\{0,1\}^* \setminus \{0^{2n}1^n \mid n \geq 0\}$

[Hint: Extend the previous grammar. What's missing?]

9. $\{w \in \{0,1\}^* \mid \#(0,w) = 2 \cdot \#(1,w)\}$ — Binary strings where the number of 0s is exactly twice the number of 1s.

10. $\{0,1\}^ \setminus \{ww \mid w \in \{0,1\}^*\}$.

[Anti-hint: The language $\{ww \mid w \in \{0,1\}^*\}$ is **not** context-free. Thus, the complement of a context-free language is not necessarily context-free!]