Rice's Theorem. *Let $\mathcal{L}$ be any set of languages that satisfies the following conditions:*

- *There is a Turing machine Y such that $\text{Accept}(Y) \in \mathcal{L}$.*
- *There is a Turing machine N such that $\text{Accept}(N) \notin \mathcal{L}$.*

The language $\text{AcceptIn}(\mathcal{L}) := \{\langle M\rangle \mid \text{Accept}(M) \in \mathcal{L}\}$ *is undecidable.*

Prove that the following languages are undecidable *using Rice's Theorem*:

1. $\text{AcceptRegular} := \{\langle M\rangle \mid \text{Accept}(M) \text{ is regular}\}$
2. $\text{AcceptIllini} := \{\langle M\rangle \mid M \text{ accepts the string } \texttt{ILLINI}\}$
3. $\text{AcceptPalindrome} := \{\langle M\rangle \mid M \text{ accepts at least one palindrome}\}$
4. $\text{AcceptThree} := \{\langle M\rangle \mid M \text{ accepts exactly three strings}\}$
5. $\text{AcceptUndecidable} := \{\langle M\rangle \mid \text{Accept}(M) \text{ is undecidable}\}$

To think about later. Which of the following are undecidable? How would you prove that?

1. $\text{Accept}\{\{\varepsilon\}\} := \{\langle M\rangle \mid M \text{ accepts only the string } \varepsilon\text{; that is, } \text{Accept}(M) = \{\varepsilon\}\}$
2. $\text{Accept}\{\varnothing\} := \{\langle M\rangle \mid M \text{ does not accept any strings; that is, } \text{Accept}(M) = \varnothing\}$
3. $\text{Accept=Reject} := \{\langle M\rangle \mid \text{Accept}(M) = \text{Reject}(M)\}$
4. $\text{Accept}\neq\text{Reject} := \{\langle M\rangle \mid \text{Accept}(M) \neq \text{Reject}(M)\}$
5. $\text{Accept}\cup\text{Reject} := \{\langle M\rangle \mid \text{Accept}(M) \cup \text{Reject}(M) = \Sigma^*\}$