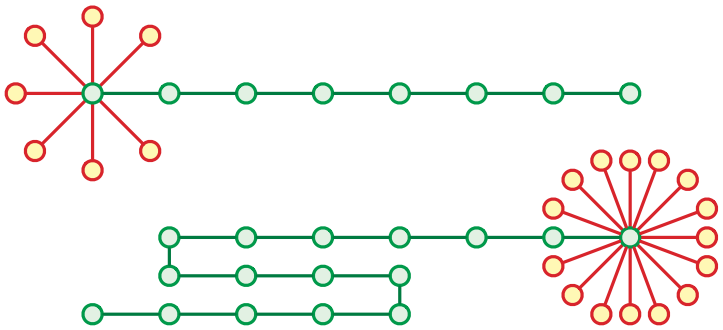
1. BALANCED 3COLOR: Suppose we are given a graph G with $3n$ vertices, for some integer n. Prove that it is NP-hard to decide whether it is possible to color each vertex of G with three colors, so that no edge connects two vertices of the same color, and there are exactly n vertices of each color.

2. LONGEST DANDELION: A *dandelion of length* ℓ consists of a path of length ℓ, with exactly ℓ new edges attached to one end. Prove that it is NP-hard to find the longest dandelion subgraph of a given undirected graph.



Two dandelions, one of length 7 and the other of length 15.

3. HIGH-DEGREE INDEPENDENT SET: Suppose we are given a graph G and an integer k. Prove that it is NP-hard to decide whether G contains an independent set of k vertices, each of which has degree at least k.

   [Hint: Reduce from the ***decision*** *version of the* INDEPENDENTSET *problem: Given a graph G and an integer k, does G contain an independent set of size k?]*

4. HALF-CLIQUE: Suppose we are given a graph G with $2n$ vertices, for some integer n. Prove that it is NP-hard to decide whether G contains a complete subgraph with n vertices?

   [Hint: Reduce from the ***decision*** *version of the* CLIQUE *problem: Given a graph G and an integer k, does G contain a clique of size k?]*