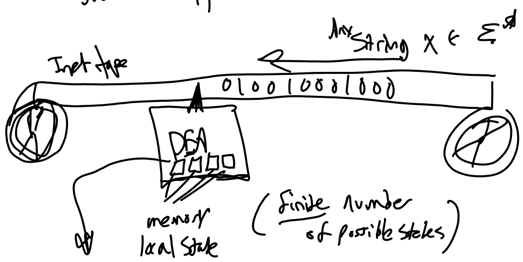
DFAs

Tuesday, September 1, 2020 9:30 AM

Deterministic Finite Automata.

I. High level approach

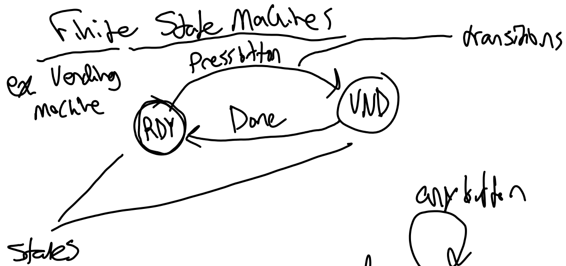


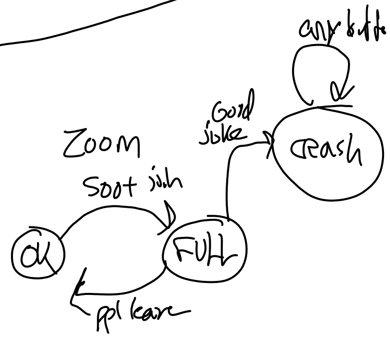
output 0 or 1 iff $x \in L$

$x \notin L$

II. Graphical view of DFA.

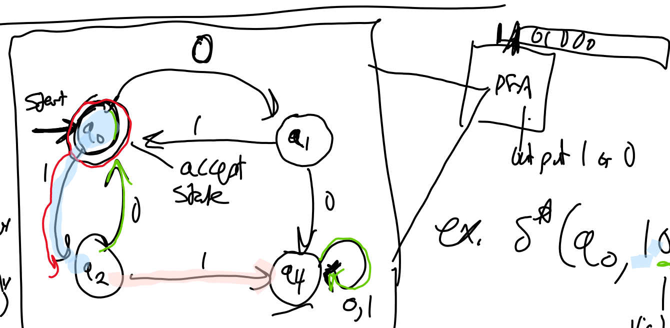
Finite State Machines





DFAs

- states
- transitions
  one for every state & every symbol
- start (only 1)
- accept states (multiple)



ex. $\delta^*(q_0, 10110)$? $= q_4$

visited the accept state

RE: $(01+10)^*$

Skill: - Trace a DFA.

Iterating function $\delta^*(q, x)$ — state, string in Σ^*

for each symbol in x, step to the next state.

- Interpreting the DFA

What language it recognizes?

If M is a DFA

$L(M)$ = the language that M recognizes

$= \{ x \in \Sigma^* \mid \delta^*(s, x) \in A \}$ ← accepting states

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III Formal Defn

A DFA M is $(Q, \Sigma, \delta, s \in Q, A)$

- Q: set of states
- Σ: alphabet
- δ: transition function
- $s \in Q$: start state
- A: accept states

ex.

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{0, 1\}$

$\delta : (Q \times \Sigma) \to Q$

- δ: single step transition
- Q: every state
- Σ: every symbol

~~$\delta(q, a) = \{ \; q = q_0 \quad q = q_1$~~

δ^* multistep entire string transitions

$\delta(q_0, 0) = q_1$

$\delta(q_0, 1) = q_2$

$\delta(q_1, 0) = q_3$

$\delta(q_1, 1) = q_1$

.....

$s = q_0$ $\quad A = \{q_0\}$

Iteration function δ^* $\qquad \delta : Q \times \Sigma \to Q$

$\delta^* : Q \times \Sigma^*$

$$\delta^*(q, x) = \begin{cases} q & x = \epsilon \\ \delta^*(\underbrace{\delta(q, a)}_{q' \text{ after one step}}, \underbrace{w}_{\text{rest of string}}) & x = a \cdot w \end{cases}$$

Inductive proofs:

Lemma: $\delta^*(q, u \cdot v) = \delta^*(\delta^*(q, u), v)$

Proof: Induction on $|u| + |v|$

Base case: $u = \epsilon$

$$\delta^*(q, \epsilon \cdot v) = \delta^*(\delta^*(q, \epsilon), v) \checkmark$$

by δ^* base case

$\epsilon \cdot v = v$ by base case

IH: for w w/ $|w| < |u|$, lemma holds.

Inductive step: $u = a \cdot w$

$$\delta^*(q, a \cdot w \cdot v) =$$

unfolding by δ^* inductive case

$$\delta^*(\delta(q, a), w \cdot v)$$

by IH

$$\delta^*(\delta^*(\delta(q, a), w), v)$$

by folding δ^* back

$$\delta^*(\delta^*(q, a \cdot w), v) \checkmark$$

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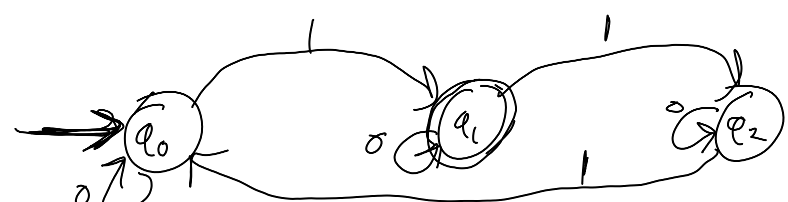
ex. # of 1's is 1 mod 3



q_i means # of 1's processed so far is i mod 3

ex. # of 1's is 1 mod 3 AND even # of 0's.

$Q = \{ q_{i,j} \mid 0 \le i < 2, \; 0 \le j < 3 \}$

$q_{i,j}$ means we've processed i mod 3 1's so far, i mod 2 0's so far

Product Construction:

D_1 and D_2 are DFAs over Σ

Product construction of D_1 and D_2, is D s.t. $L(D) = L(D_1) \cap L(D_2)$

Let $D_1 = (Q_1, \delta_1, s_1, A_1)$ $\quad\Sigma$ is same.

$D_2 = (Q_2, \delta_2, s_2, A_2)$

$Q_1 = a_1, a_2 \ldots$

$Q_2 \; p_1, p_2, \vdots \quad (a_i, p_j)$

Then: $Q = Q_1 \times Q_2 = \{ (q_1, q_2) \mid q_1 \in Q_1, \; q_2 \in Q_2 \}$

$s = (s_1, s_2)$

$\delta((q_1, q_2), a) = (\underbrace{\delta_1(q_1, a)}_{\text{something from } Q_1}, \underbrace{\delta_2(q_2, a)}_{\text{something from } Q_2})$

Correctness: $\delta^*((q_1, q_2), x) = (\delta^*(q_1, x), \delta^*(q_2, x))$

$A = \{ (q_1, q_2) : q_1 \in A_1, \; q_2 \in A_2 \}$

$= A_1 \times A_1$

$H_1 \wedge H_2$.