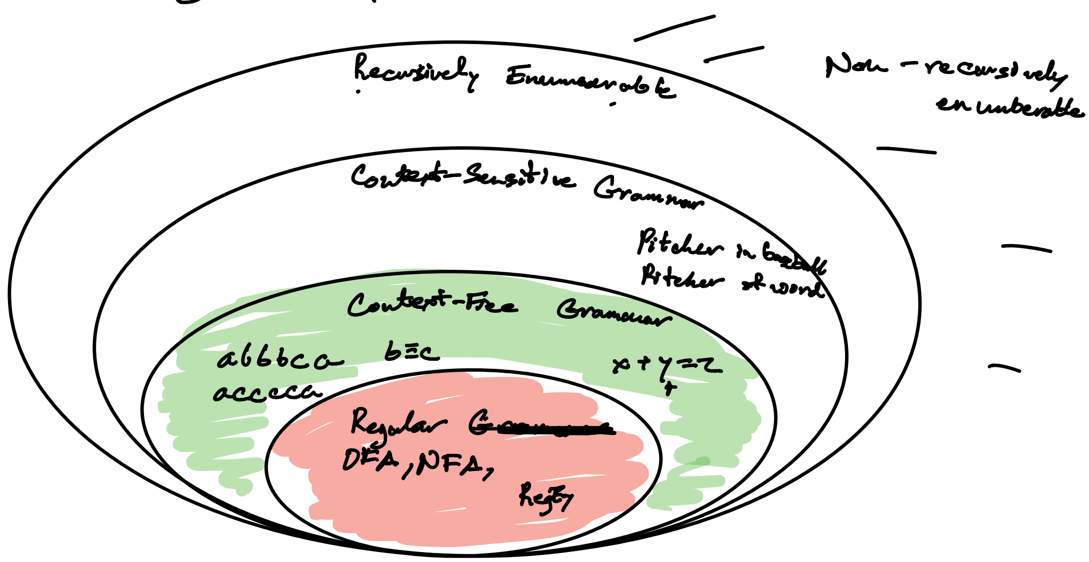
Lecture #7 - Context-Free Grammers & Languages

Chomsky-Schutzenberg Hierarchy



Regular Languages - language that can be defined by union, concatenation & repetition

$L = \{0^n 1^n \mid n \geq 0\}$

Context-Free Language → defined by union, concatenation, recursion

$G = (\Gamma, \Sigma, P, S)$

Γ set of symbols which are non-terminals

Σ set of symbols (terminals) $\quad w \quad L$

P set of production of the form $A \to w$ where $A \in \Gamma$, $w \in (\Sigma \cup \Gamma)^*$

S starting non-terminal

Example #1 $\quad \Sigma = \{0, 1\} \quad \Gamma = \{S, A, B, C\} \quad S = S$

$$P = \begin{cases} S \to A & A \to 0A & B \to B1 & C \to \varepsilon \\ S \to B & A \to 0C & B \to C1 & C \to 0C1 \end{cases}$$

$S \rightsquigarrow A \rightsquigarrow 0A \to 00A \rightsquigarrow 000A \rightsquigarrow 0000C \rightsquigarrow 0000$

$\rightsquigarrow 0000011$

$L = \{ S \overset{n}{\rightsquigarrow} \omega \mid n \geq 0 \}$ $\quad \rightsquigarrow 000000000 \in 11111$

Guess #1 : $0^* 1^*$ $\quad 00 \quad\quad \omega = 0011$

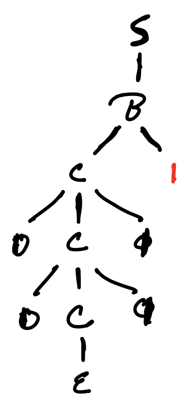
Language $\{ 0^m 0^n 1^n + 0^n 1^n 1^m \mid m, n \geq 0 \}$

Parse trees

$\Sigma = \{0, 1\} \quad \Gamma = \{S, A, B, C\} \quad S = S$

$$P = \begin{cases} S \underset{1}{\to} A & A \to 0A \quad B \to B1 \quad C \to \varepsilon \\ S \underset{2}{\to} B & A \to 0C \quad B \to C1 \quad C \to 0C1 \end{cases}$$

00011



$\omega = 00111 \quad L(G) \quad 0^m 0^n 1^n + 0^n 1^n 1^m$

Example 2 #: Parentheses Matching

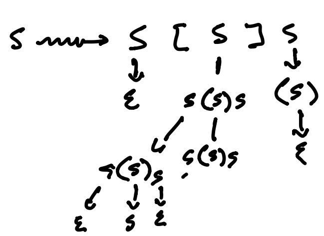
ω where all parentheses rules apply

$\omega = [()(())]() \in L_{PAR}$

$[(]))\notin L_P$

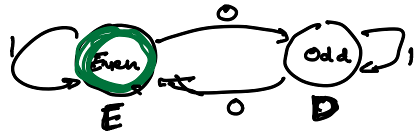
$\Sigma = \{ (,), [,] \} \quad \Gamma = \{S\} \quad S = S$

$S \to S(S)S \mid S[S]S \mid \varepsilon \quad \equiv \quad \begin{array}{l} S \to S \\ S \to (S) \\ S \to [S] \end{array}$



Example #3   Regular → CFG $\Sigma = \{0,1\}$

$L = \{\omega \mid \omega \text{ has even \# } 0\text{'s}\}$ $(1^*01^*01^*)^*$



$S = S$

$\Gamma = \{E, D, S\}$

$$P = \begin{cases} S \to E & D \to 1D \mid 0E \\ E \to 1E \mid 0D & E \to \varepsilon \end{cases}$$

1 1 0 0 1 1 0 1 0

Induction Proofs using CFGs

Consider the following grammar:

$\Sigma = \{a,b,c\}$ $\Gamma = \{S\}$ $S = S$ $P = S \to ab \mid ac \mid aSb \mid aSSc$ (1, 2, 3, 4)

Claim: $\boxed{\text{For any } \omega \in L_\Sigma \quad \# \text{ of } a\text{'s} = \# b\text{'s} + \# c\text{'s}}$

$L_\Sigma = \{ ab_1,\ aabb_{3,1},\ aacb_{3,2},\ aabacb_{4,1,2}$

All words in L_Σ satisfy $S \overset{n}{\Rightarrow} w$

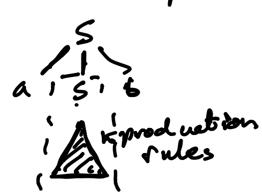
Base Claim: If $n = 1$ (only one production rule) $\omega = ab$ or ac

Inductive Hypothesis: for all $k \geq 1$, $r < k$ application of P then $\#_a = \#_b + \#_c$

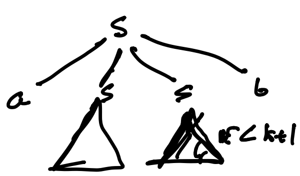
Want to prove: For $r = \underline{k+1}$ applications then holds true

If is 1 or 2 then production rule $P = S \to ab \mid ac \mid aSb \mid aSSc$ (1, 2, 3, 4)

If (3)

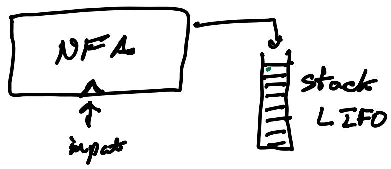


If (4)

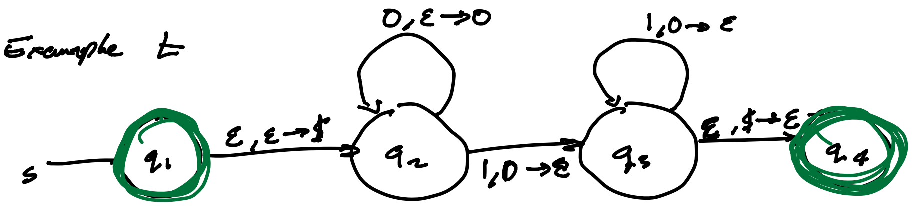


Claim holds true

Pushdown Automata:



Example L



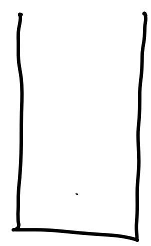
s, + → +

input

poped stack value

pushed stacked value

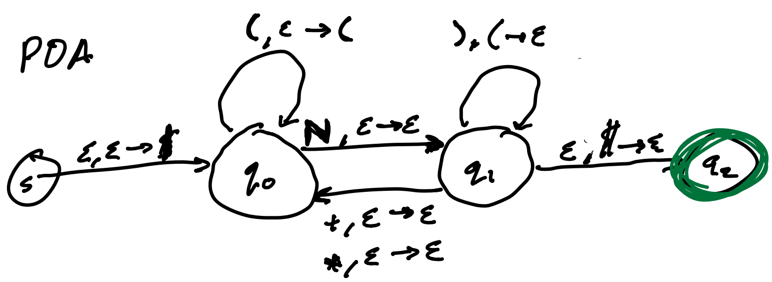
0011



$\omega = 0011$

$L = \{0^n 1^n \mid n \geq 0\}$

PDA



$\Sigma = \{ (,), +, *, \mathbb{N} \}$ $\Gamma = \{ S, q_0, q_1, q_2 \}$ $S = S$

((2+3)*6) Accepted

((2*)