Lecture 26 Scribble

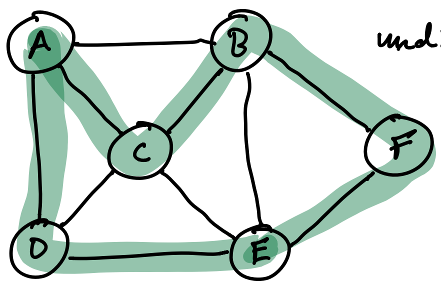
Chat Moderators: Eliot & Tanvi

Topics: 3SAT to Hamiltonian Cycle

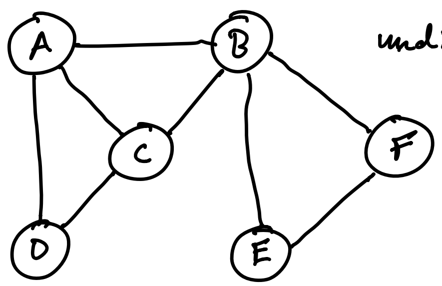
3SAT to 3-Color

3SAT to CSAT

Hamiltonian Cycle?



undirected graph



undirected graph

Problem P1:

Input: Given a graph $G=(V,E)$ with $|V|=n$ vertices

Goal: Does G have a hamiltonian cycle

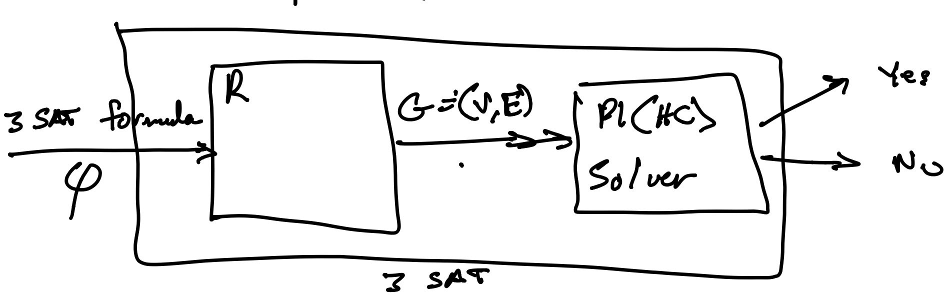
Posit that P1 is NP-complete.

1. P1 must be in NP? Yes
   Certificate → list of vertices

2. P1 is NP-hard?

   We know $L \in NP \leq_p 3SAT$

   So if $3SAT \leq_p P1\ (HC)$ then P1 is NP-hard



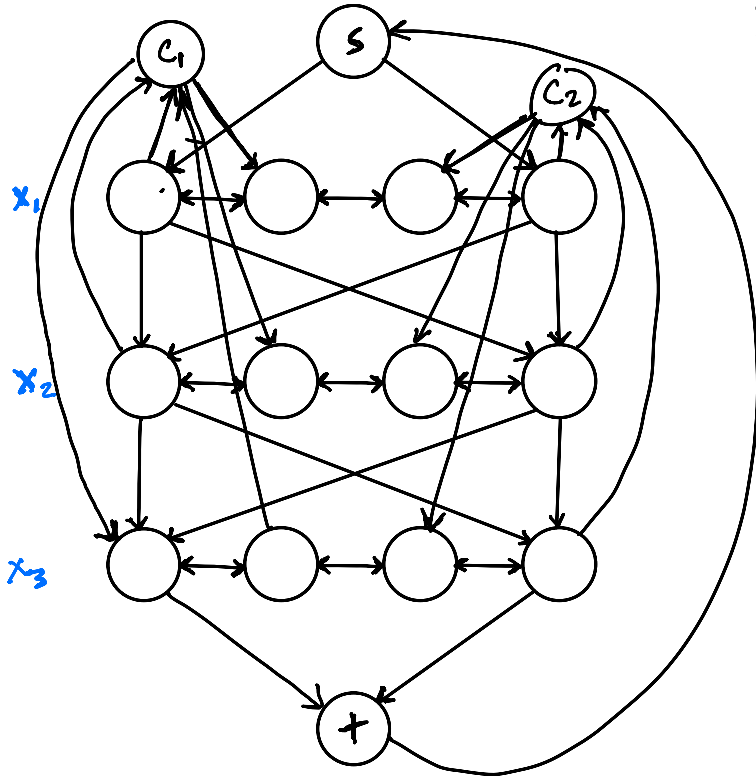
Convert φ to G with HC?

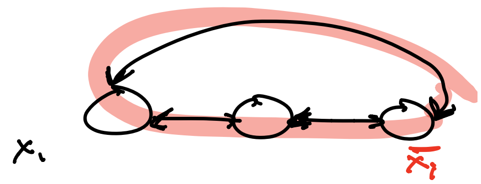
φ has n variables
m clauses

$\varphi = ?$

$x_1 \quad \bar{x}_2 \quad x_3 \longrightarrow G(V, E)$ hamiltonian cycle

Left to Right = True
Right to Left = False





$x = \langle 1, 0, 1 \rangle$

$x = \langle 0, 0, 1 \rangle$

$$\varphi = \underbrace{(x_1)}_{C_1}$$

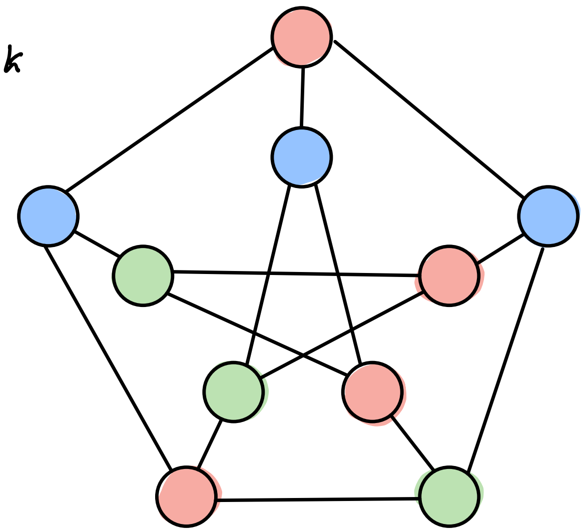
$$\varphi = (x_1 \vee x_2 \vee \bar{x}_3)$$

$$\varphi = (x_1 \vee x_2 \vee \bar{x}_3) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3)$$

Graph Coloring

Given a (undirected) graph $G=(V,E)$ and integer k

Can the vertices of G be colored using k colors so all vertices connected by an edge have different colors



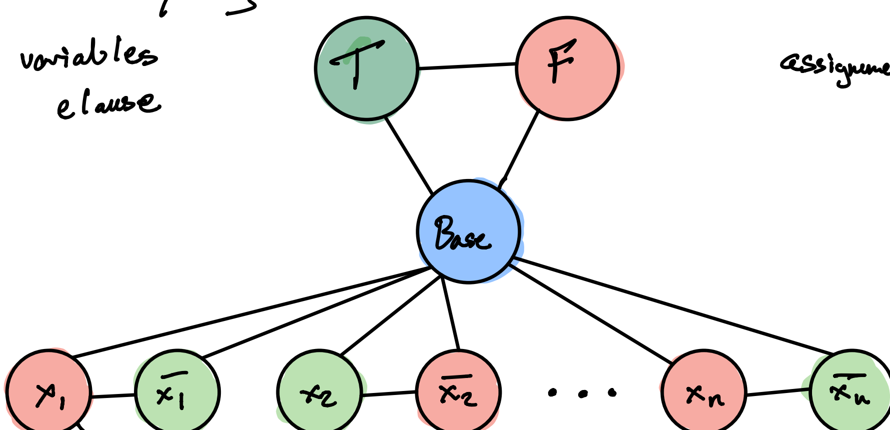
3SAT to 3-Coloring

φ = anything

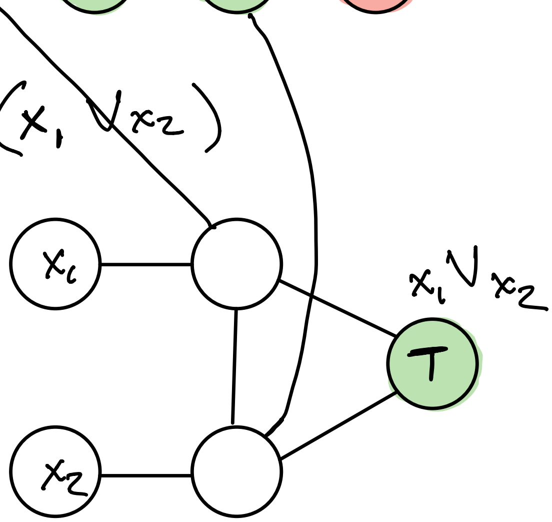
n variables

m clause

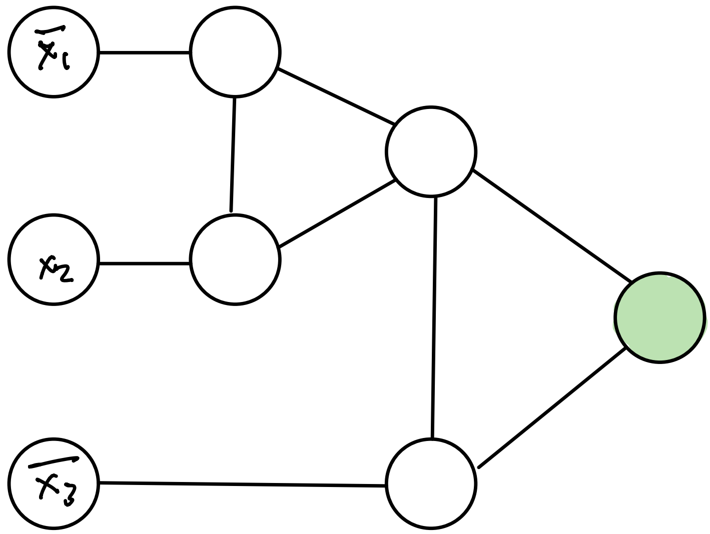
assignment = $x_1 \; x_2 \; x_3$ / $\bar{x}_1 \; \bar{x}_2 \; \bar{x}_3$



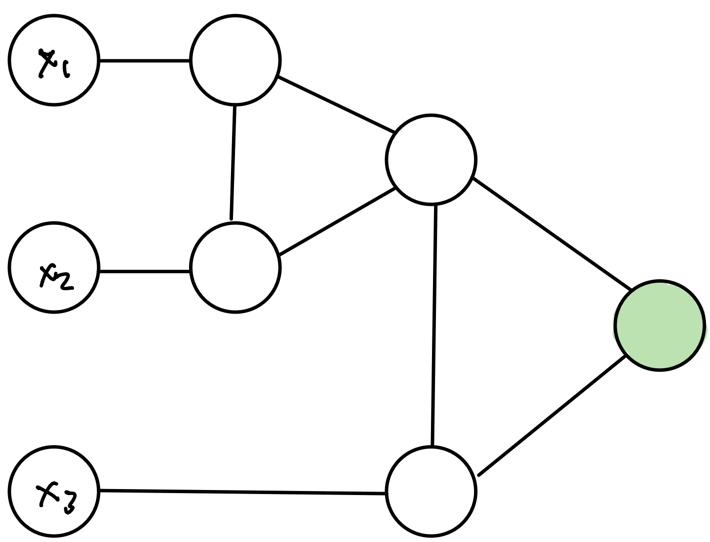
$\varphi = (x_1 \vee x_2)$



$$\varphi = (\overline{x_1} \vee x_2 \vee \overline{x_3})$$

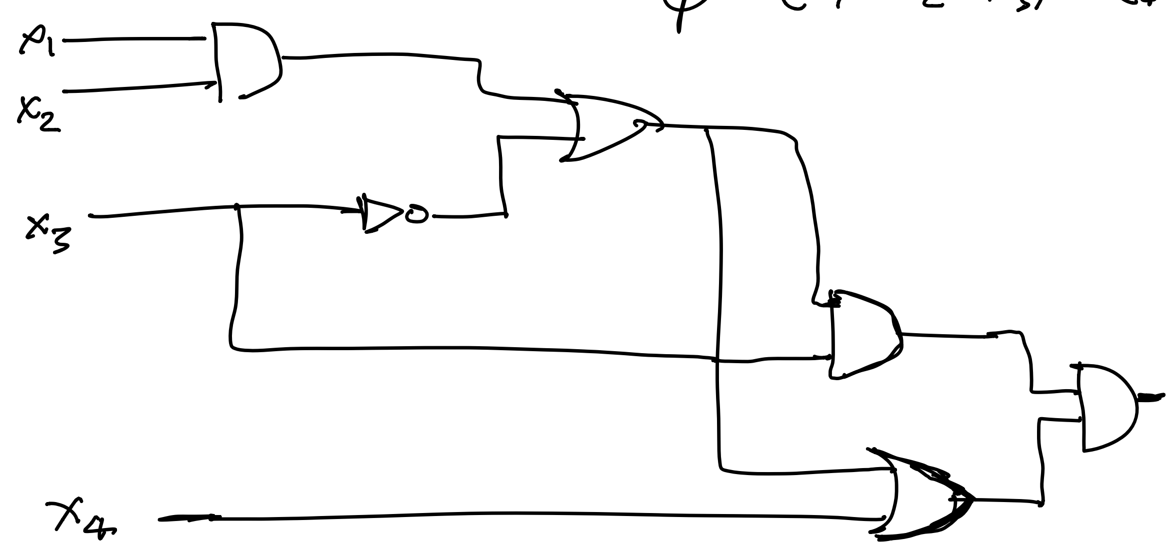


$$\varphi = (\overline{x_1} \vee x_2 \vee \overline{x_3}) \vee (x_1 \vee x_2 \vee x_3)$$



Circuit SAT

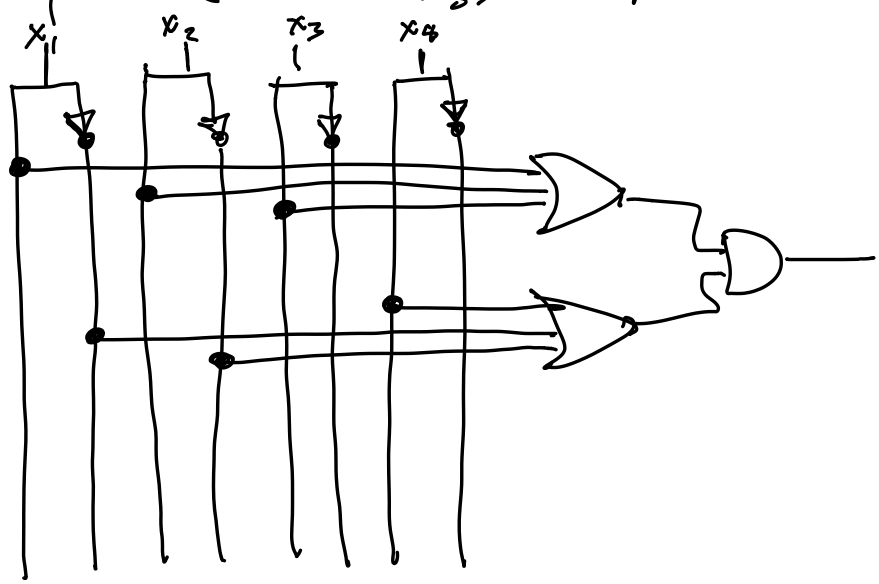
$\phi = (x_1 \vee x_2 \vee x_3) \wedge (x_4 \ldots$



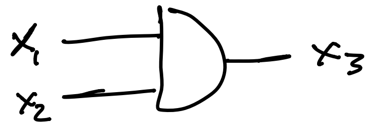
Given a circuit, is there an assignment that outputs true

3SAT $\leq_p$ Circuit SAT

$\phi = (x_1 \vee x_2 \vee x_3) \wedge (x_4 \vee \bar{x}_1 \vee \bar{x}_2)$



Circuit SAT $\leq_p$ 3SAT



$x_3 = x_1 \land x_2$

x_3	x_1	x_2	$x_3 = x_1 \land x_2$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

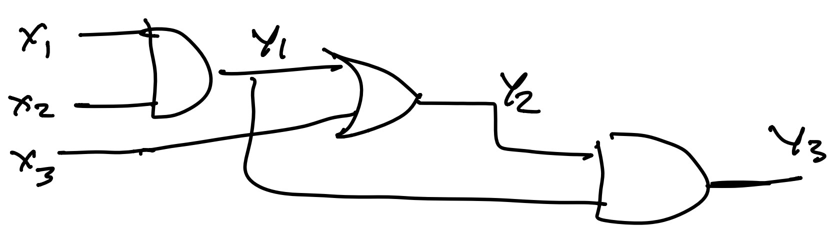
CNF

$$\varphi_{AND} = (z \lor \bar{x} \lor \bar{y}) \land (\bar{z} \lor x \lor y) \land (\bar{z} \lor x \lor \bar{y}) \land (\bar{z} \lor \bar{x} \lor y) \land x_3$$

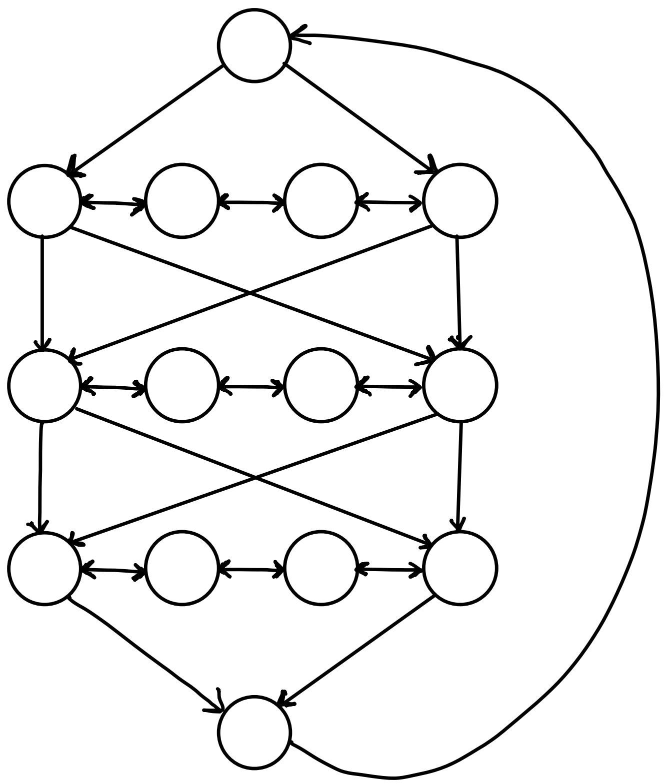
DNF

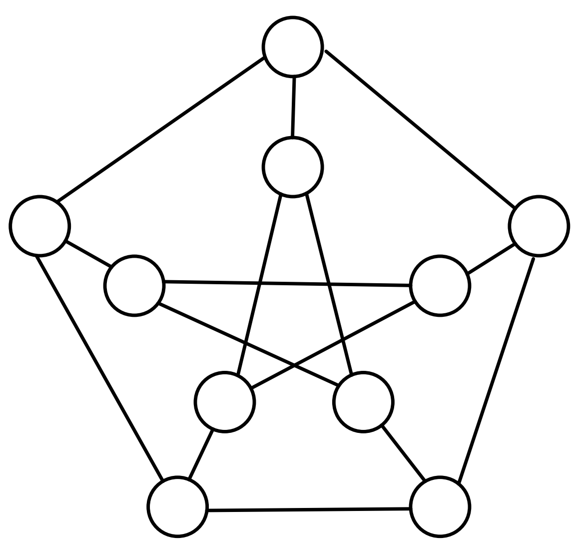
$$\beta = (\bar{x}_1 \cdot \bar{x}_2 \cdot \bar{x}_3) + (\ldots$$

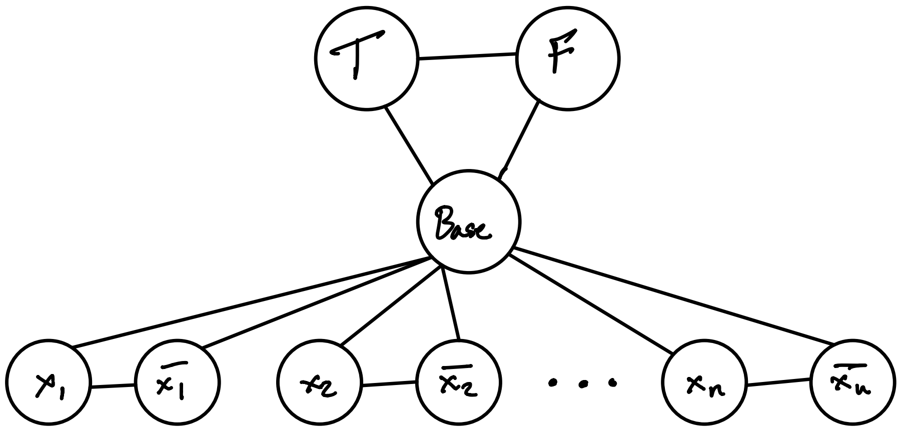
$\vdots$



$$\varphi = y_3 \land (y_2 = y_1 \lor x_3) \land (y_1 = x_1 \land x_2)$$






T
F
Base
x_1
$\overline{x_1}$
x_2
$\overline{x_2}$
x_n
$\overline{x_n}$