Lecture 10 Scribble - Midterm # 1 Review

Chat moderators: Tanvi & Vasilis

Topics to cover:

- Reductions
- Recurrences
- Context-Free Grammer
- DFA formal problems

$E_{TM} = \{\langle M\rangle \mid M \text{ is a TM } \& \ L(M) = \emptyset\}$ is undecidable

Reduction Example #1:

$EQ_{TM} = \{\langle M_1, M_2\rangle \mid M_1 \text{ and } M_2 \text{ are TMs } \& \ L(M_1) = L(M_2)\}$

Prove EQ_{TM} is undecidable.

Proof by contradiction

$$R = \begin{cases} \text{accepts} & \text{if } L(M_1) = L(M_2) \\ \text{rejects} & \text{if } L(M_1) \neq L(M_2) \end{cases}$$

Assume a TM M_2 that always rejects; $L(M_2) = \emptyset$

S = "On input $\langle M_1\rangle$ where M_1 is a TM

1. Run R on input $\langle M_1, M_2\rangle$
2. If R accepts, S accepts. If R rejects, S rejects"

EQ_{TM} is undecidable.

$A_{TM} \{ \langle M, w \rangle \mid M$ is a TM, w is a string and M accepts $w \}$

We know A_{TM} is undecidable

Reduction Problem #2

$Regular_{TM} = \{ \langle M \rangle \mid M$ is a TM and $L(M)$ is a regular language$\}$

Prove $Regular_{TM}$ is undecidable.

Proof by Contradiction:

Assume $Regular_{TM}$ is decidable:

$$R(\langle M_2 \rangle) = \begin{cases} \text{accepts} & \text{if } L(M_2) \text{ is regular} \\ \text{rejects} & \text{if } L(M_2) \text{ is not regular} \end{cases}$$

If M_2 dna. x $L(M_2) = 0^n 1^n$ (not regular) R rejects

$M_2(x) =$ " On input x

If M_2 acc x $L(M_2) = \Sigma^*$ (regular) R accepts

If M dna w $L(M_2)$

1. If x has the form $0^n 1^n 2^n$, accept
2. If x has any other form, then run M on w and accept if M accepts w"

$$R(M_2) = \begin{array}{l} \text{reject if } L(M_2) \text{ is not regular} \\ \text{accept if } L(M_2) \text{ is regular} \end{array}$$

$S =$ " 1. Construct TM M_2

2. Run $R(\langle M_2 \rangle)$

3. If R accepts, accept . . . . "

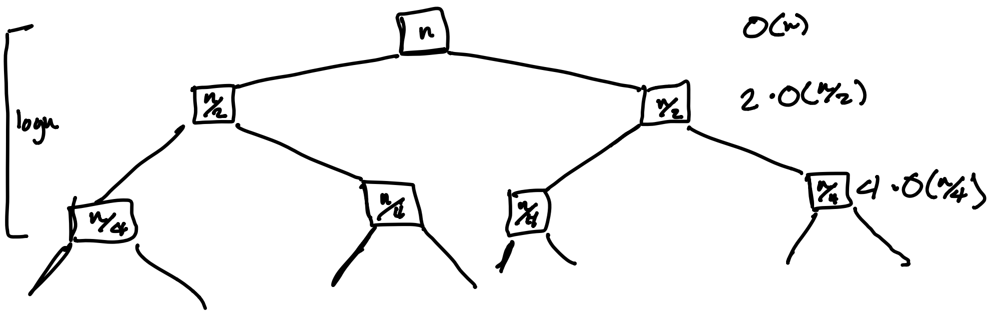
S decides A_{TM}, we know A_{TM} is undecidable.

$Regular_{TM}$ is undecidable.

Recurrences

$O(n) = k\, n$

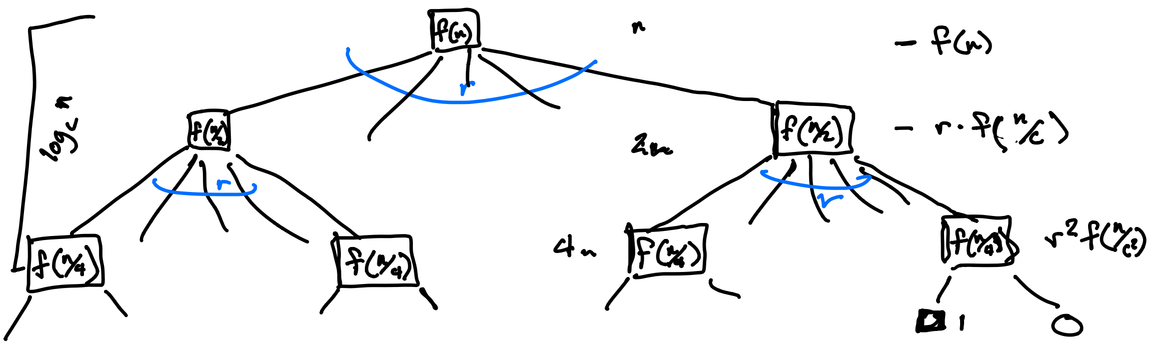
$T(n) = 2T(n/2) + O(n)$



$T(n) = O(n \log n) \neq n \log n$

Generalize Recurrence

$T(n) = r\, T(n/c) + f(n)$



If $f(n) > r \cdot f(n/c) > r^2 f(n/c^2) > \ldots\ldots$

$T(n) = O(f(n))$

If $f(n) = r \cdot f(n/c) = r^2 f(n/c^2) = \ldots\ldots$

$T(n) = O(\log_c n \cdot f(n))$

If $f(n) < r \cdot f(n/c) < r^2 f(n/c^2) < \ldots\ldots$

$r^{\log_c n} = n^{\log_c r}$

$T(n) = O(1 \cdot n^{\log_c r})$

Context Free Grammar:

$L = \{a^i b^j c^k \mid i \le j+k\}$

Describe the CFG that gives this language.

First solve simpler case

$L_x = \{a^i b^j c^k \mid i,j,k \geq 0\}$

$S \to aS \mid Sc \mid Sb \mid \varepsilon$

$aS \rightsquigarrow aSb \rightsquigarrow aScb \rightsquigarrow \varepsilon$

$S \to aS \mid Sc \mid B$

$B \to Bb \mid \varepsilon$

$L = \{a^i b^j c^k \mid i \le j+k\}$

$S \to aSc \mid Sc \mid B$

$B \to aBb \mid Bb \mid \varepsilon$