A list of useful NP-Complete problems

1 Satisfiability

Circuit Satisfiability

Instance: A circuit C with m inputs
Question: Is there an input for C such that C returns true for it.

Definition 1.1 A boolean formula is in conjunctive normal form (***CNF***) if it is a conjunction (AND) of several *clauses*, where a clause is the disjunction (or) of several *literals*. A literal is either a variable or a negation of a variable.

SAT

Instance: A CNF formula F with n variables
Question: Is there an assignment to the variables such that F is TRUE?

Definition 1.2 ***3CNF*** formula is a CNF formula with *exactly* three literals in each clause.

S3AT

Instance: A 3CNF formula F with n variables
Question: Is there an assignment to the variables such that F is TRUE?

2 Clique/independent set/vertex cover

Definition 2.1 A ***clique*** is a complete graph, where every pair of vertices are connected by an edge.

Clique

Instance: A graph G, integer k
Question: Is there a subgraph H in G with k vertices, such that H is a clique?

Definition 2.2 A set S of nodes in a graph $\mathsf{G} = (V, E)$ is an ***independent set***, if no pair of vertices in S are connected by an edge.

Independent Set

Instance: A graph G, integer k
Question: Is there an independent set in G of size k?

Definition 2.3 For a graph G, a set of vertices $S \subseteq V(\mathsf{G})$ is a ***vertex cover*** if it touches every *edge* of G.

Vertex Cover

Instance: A graph G, integer k **Question:** Is there a vertex cover in G of size k?

3 Coloring

Definition 3.1 A ***coloring***, by c colors, of a graph $\mathsf{G} = (V, E)$ is a mapping $C : V(\mathsf{G}) \rightarrow \{1, 2, \ldots, c\}$ such that every vertex is assigned a color, such that no two vertices that share an edge are assigned the same color.

3Colorable

Instance: A graph G. **Question:** Is there a coloring of G using three colors?

4 Hamiltonian paths/cycles and TSP

Definition 4.1 A ***Hamiltonian cycle*** is a cycle in the graph that visits every vertex exactly once.

Hamiltonian Cycle

Instance: A graph G. **Question:** Is there a Hamiltonian cycle in G?

Hamiltonian Cycle is NP-Complete both for directed and undirected graphs.

Hamiltonian Path

Instance: A graph G. **Question:** Is there a Hamiltonian path in G? Namely, is there a simple path that visits all the vertices of G exactly once.

Hamiltonian Path is NP-Complete both for directed and undirected graphs. It remains NPC even if you specify the start and end vertices of the path.

Definition 4.2 A ***traveling salesman tour*** (***TSP***), is a Hamiltonian cycle in a graph. Its price is the total price of all its edges.

TSP

Instance: $G = (V, E)$ a complete graph - n vertices, $c(e)$: Integer cost function over the edges of G, and k an integer. **Question:** Is there a traveling-salesman tour with cost at most k?

TSP remains NP-Complete if the graph directed/undirected or if instead of a closed tour, one is looking for a path that visits every vertex exactly once.

5 Subset sum and partition

Subset Sum

Instance: S - set of positive integers,t: - an integer number (Target)
Question: Is there a subset $X \subseteq S$ such that $\sum_{x \in X} x = t$?

Partition

Instance: A set S of n numbers.
Question: Is there a subset $T \subseteq S$ s.t. $\sum_{t \in T} t = \sum_{s \in S \setminus T} s$?

6 Three dimensional matching and set cover

3DM

Instance: X, Y, Z sets of n elements, and T a set of triples, such that $(a, b, c) \in T \subseteq X \times Y \times Z$.
Question: Is there a subset $S \subseteq T$ of n disjoint triples, s.t. every element of $X \cup Y \cup Z$ is covered exactly once.?

SET COVER

Instance: $(\mathsf{U}, \mathcal{F}, k)$:
U: A set of n elements
$\mathcal{F}$: A family of subsets of U, s.t. $\bigcup_{X \in \mathcal{F}} X = \mathsf{U}$.
k: A positive integer.
Question: Are there k sets $S_1, \ldots, S_k \in \mathcal{F}$ that cover U. Formally, $\bigcup_i S_i = \mathsf{U}$?